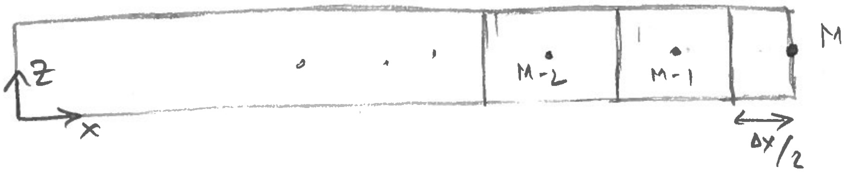
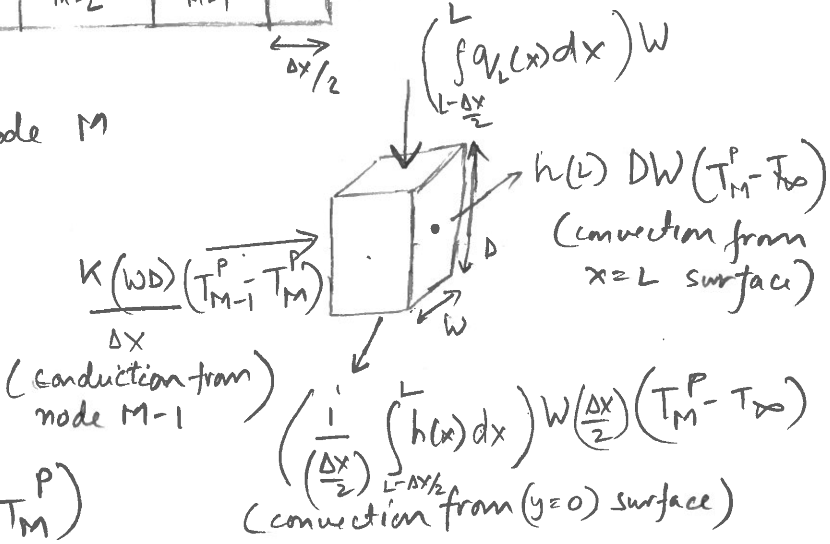
Problem 1



Energy balance for node M

T_M^p ← time index
↖ space index



Storage rate $= \rho C\left(\frac{\Delta x}{2}\right)WD\frac{\left(T_M^{p+1} - T_M^p\right)}{\Delta t}$

{ z=0 and z=W surfaces are insulated }.

From energy balance, $\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out}$

$$\Rightarrow \boxed{\rho C\left(\frac{\Delta x}{2}\right)WD\frac{\left(T_M^{p+1} - T_M^p\right)}{\Delta t} = \frac{K(WD)}{\Delta x}\left(T_{M-1}^p - T_M^p\right) + W\int_{L-\frac{\Delta x}{2}}^{L} q_L(x)\,dx - h(L)DW\left(T_M^p - T_\infty\right) - \left(T_M^p - T_\infty\right)W\int_{L-\frac{\Delta x}{2}}^{L} h(x)\,dx}$$

PROBLEM 2

$$T_{REF} = \frac{T_s + T_\infty}{2} = \frac{200K + 100K}{2} = 150K$$

FOR AIR AT 150K, $\nu = 4.426 \times 10^{-6} \frac{m^2}{s}$, $Pr = 0.751$, $K = 0.0138 \frac{W}{m \cdot K}$

a) LIMITS OF $10 \frac{m}{s}$ AND $1000 \frac{m}{s}$

AT FLOW OF $10 \frac{m}{s}$, $Re_x = \frac{U_\infty X_s}{\nu} = \frac{10 \frac{m}{s} \cdot 0.5m}{4.426 \times 10^{-6} \frac{m^2}{s}} = 1.13 \times 10^6$

AT FLOW OF $1000 \frac{m}{s}$, $Re_x = \frac{U_\infty X_s}{\nu} = \frac{1000 \frac{m}{s} \cdot 0.5m}{4.426 \times 10^{-6} \frac{m^2}{s}} = 1.13 \times 10^8$

AT BOTH SPECIFIED LIMITS, FLOW WILL BE TURBULENT, THEREFORE, DEVICE OPERATES IN TURBULENT REGIME

b) $q''(x_s) = 20,000 \frac{W}{m^2} = h(T_s - T_\infty) = q_{conv}$

$$20,000 \frac{W}{m^2} = h(200 - 100) \Rightarrow h = 200 \frac{W}{m^2 \cdot K}$$

WITH TURBULENT FLOW, Pr = 0.751, AND WANTING A LOCAL NUSSELT NUMBER, WE SELECT THE LAST GIVEN CORRELATION (Re_x SHOULD BE IN RANGE, BASED ON PART a)

$$Nu_x = 0.0296\, Re_x^{4/5}\, Pr^{1/3}$$

$$\Rightarrow \left(\frac{Nu_x}{0.0296\, Pr^{1/3}}\right)^{5/4} = Re_x = \frac{U_\infty X_s}{\nu}$$ (NOTING THAT $Nu_x = \frac{hx}{K}$)

$$\Rightarrow U_\infty = \frac{\nu}{X_s}\left(\frac{h\, x_s}{0.0296 \cdot Pr^{1/3} \cdot K}\right)^{5/4}$$

$$U_\infty = \frac{4.426 \times 10^{-6} \frac{m^2}{s}}{0.5m}\left(\frac{200 \frac{W}{m^2 \cdot K} \cdot 0.5\, m}{0.0296 \cdot Pr^{1/3} \cdot 0.0138 \frac{W}{m \cdot K}}\right)^{5/4} = \boxed{54.101 \frac{m}{s} = U_\infty}$$

Problem 3

$$T(x,y) - T_\infty = (T_s - T_\infty)\exp\left(-\frac{y}{\gamma x^{1/4}}\right) \Rightarrow \frac{T(x,y)-T_\infty}{T_s - T_\infty} = \exp\left(\frac{-y}{\gamma x^{1/4}}\right)$$

(a) @ $y = \delta_T$, $\dfrac{T(x,\delta_T) - T_\infty}{T_s - T_\infty} = 0.01$

(definition of thermal B.L)

$$\Rightarrow \quad 0.01 = \exp\left(-\frac{\delta_T(x)}{\gamma x^{1/4}}\right)$$

$$\Rightarrow \quad \frac{\delta_T}{\gamma x^{1/4}} = -\ln(0.01) = 4.61 \Rightarrow \boxed{\delta_T = 4.61\,\gamma x^{1/4}}$$

(b) $$h(T_s - T_\infty) \doteq -k\left.\frac{\partial T}{\partial y}\right|_{y=0} = -(T_s - T_\infty)\frac{\partial}{\partial y}\left.\left(\exp\left(-\frac{y}{\gamma x^{1/4}}\right)\right)\right|_{y=0}$$

$$= -k(T_s - T_\infty)\left[\exp\left(-\frac{y}{\gamma x^{1/4}}\right)\cdot\left(-\frac{1}{\gamma x^{1/4}}\right)\right]_{y=0}$$

$$\Rightarrow h\cancel{(T_s - T_\infty)} = \frac{k\cancel{(T_s - T_\infty)}}{\gamma x^{1/4}}$$

$$\boxed{Nu_x = \frac{hx}{k} = \frac{x}{\gamma x^{1/4}} = \frac{x^{3/4}}{\gamma}}$$

(c) $$Nu_x = \frac{x^{3/4}}{\left(\dfrac{10\,\nu^{1/2}\alpha^{1/4}}{U_\infty^{3/4}}\right)} = 0.1\,(U_\infty x)^{3/4} \times \left(\frac{\nu}{\alpha}\right)^{1/4} \times \frac{1}{\nu^{3/4}}$$

$$= 0.1\left(\frac{U_\infty x}{\nu}\right)^{3/4}\left(\frac{\nu}{\alpha}\right)^{1/4}$$

$$\boxed{Nu_x = 0.1\,(Re_x)^{3/4}\,Pr^{1/4}}$$