PROBLEM 1

HEAT EQN: $$\frac{\partial^2 T}{\partial x^2} + \underbrace{\cancel{\frac{\partial^2 T}{\partial y^2}}}_{0} + \underbrace{\cancel{\frac{\partial^2 T}{\partial z^2}}}_{0} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\underbrace{\cancel{\frac{\partial T}{\partial t}}}_{0,\ SS}$$

0, NO y, z GRADIENTS

$$\rightarrow \frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = 0$$

BC: ① @ $x = 0$ $\quad T = T_\infty$

② @ $x = L$ $\quad -k\frac{\partial T}{\partial x} = h\left(T(x=L) - T_\infty\right)$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\gamma \sin\left(\frac{\pi x}{L}\right)}{k} = 0$$

INTEGRATE

$$\frac{\partial T}{\partial x} - \frac{L}{\pi}\frac{\gamma\cos\left(\frac{\pi x}{L}\right)}{k} + C_1 = 0 \quad \text{OR} \quad \frac{\partial T}{\partial x} = \frac{L}{\pi}\frac{\gamma\cos\left(\frac{\pi x}{L}\right)}{k} + C_1$$

INTEGRATE

$$T - \left(\frac{L}{\pi}\right)^2\frac{\gamma}{k}\sin\left(\frac{\pi x}{L}\right) + C_1 x + C_2 = 0 \quad \text{OR} \quad T = \left(\frac{L}{\pi}\right)^2\frac{\gamma}{k}\sin\left(\frac{\pi x}{L}\right) + C_1 x + C_2$$

APPLY BC ①

$$T = \frac{L^2\gamma}{\pi^2 k}\underbrace{\cancel{\sin(0)}}_{0} + C_1\underbrace{\cancel{(0)}}_{0} + C_2 = T_\infty \Rightarrow \boxed{C_2 = T_\infty}$$

APPLY BC ②

$$-k\left[\frac{\partial T}{\partial x} = \frac{L\gamma}{\pi k}\cos\left(\frac{\pi x}{K}\right) + C_1\right]_{x=L} = h\left(T(x=L) - T_\infty\right) = h\left[\left(\frac{L}{\pi}\right)^2\frac{\gamma}{k}\sin\left(\frac{\pi x}{L}\right) + C_1 x + C_2 - T_\infty\right]_{x=}$$

$$-k\frac{\partial T}{\partial x} = -\cancel{k}\frac{L\gamma}{\pi\cancel{k}}\underbrace{\cancel{\cos\left(\frac{\pi L}{L}\right)}}_{-1} - kC_1 = h\left(\left(\frac{L}{\pi}\right)^2\frac{\gamma}{k}\underbrace{\cancel{\sin\left(\frac{\pi L}{L}\right)}}_{0} + C_1 L + \cancel{C_2} - \cancel{T_\infty}\right)$$

SINCE $C_2 = T_\infty$

$$\frac{L\gamma}{\pi} - kC_1 = hC_1 L$$

$$\frac{L\gamma}{\pi} = C_1(hL + k)$$

$$\boxed{C_1 = \frac{L\gamma}{\pi(hL+K)}}$$

$$\Rightarrow \boxed{T(x) = \left(\frac{L}{\pi}\right)^2\frac{\gamma}{k}\sin\left(\frac{\pi x}{L}\right) + \frac{\gamma L x}{\pi(hL+k)} + T_\infty}$$

2

(a) Let's first find the parameter, $m = \sqrt{\frac{hP}{KA_c}}$, for the fin

$P = 2\pi R$; $\quad A_c \approx 2\pi R \delta$ (using $\delta << R$ approximation)

$$m = \sqrt{\frac{hP}{KA_c}} = \sqrt{\frac{h \cdot 2\pi R}{K \cdot 2\pi R \delta}} = \sqrt{\frac{h}{K\delta}} = \sqrt{\frac{10\,[W/m^2K]}{15\,[W/mK]\, 1\times10^{-3}\,[m]}}$$

$\Rightarrow m = 25.8 \; m^{-1}$

For this fin, $mL = 25.8\,[m^{-1}] \times 250\times10^{-3}\,[m]$

$= 6.45 > 2.65$

(infinitely long fin approx. is valid)

$$T(x = 100\,mm) = T_\infty + (T_b - T_\infty)\, e^{-mx}$$

$$= 20^\circ C + (220^\circ C - 20^\circ C)\, e^{-25.8\,[m^{-1}]\times 0.1\,[m]}$$

$$\approx \boxed{35.2^\circ C}$$

(b) For infinitely long fin, $q = \sqrt{hPKA}\,(T_b - T_\infty)$

$= \sqrt{h\, 2\pi R\, K\, 2\pi R \delta}\,(T_b - T_\infty)$

$= 2\pi R \sqrt{hK\delta}\,(T_b - T_\infty)$

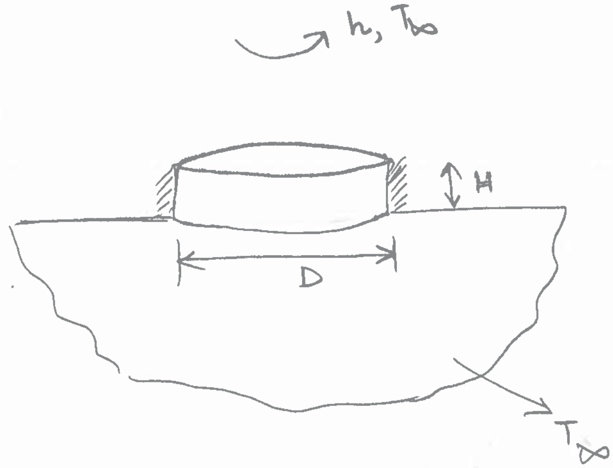
(i) Reducing L by 10%, reduces mL to $0.9 \times 6.45 = 5.8 > 2.65$.

$\rightarrow$ The fin is still infinite. Thus reducing L does not affect q_f

(ii) & (iii) From the formula for q_f, we can conclude that reducing R by 10%, reduces q by 10%, while reducing δ by 10% reduces q by 10%/2 = 5%. $\boxed{\text{Option (ii) reducing } R \text{ will best reduce } q}$

Note that you could also solve the same problem using the adiabatic fin tip approach and get the same answer. Since tanh(mL) ~1 for mL > 2.65, q is equal for the "adiabatic fin tip" and the "infinitely long fin" approach.

$\boxed{3}$



we know that $Bi = \dfrac{\Delta T_{internal}}{\Delta T_{external}}$

$= \dfrac{R_{int}}{R_{ext}}$.

Let's first consider a characteristic internal resistance for the hot aluminum disk.

$$R_{int} = \frac{H}{K_{Al}\left(\frac{\pi D^2}{4}\right)}$$

For R_{ext}, we recognize that there are 2 resistances in parallel,

(i) convection resistance $\rightarrow$ $R_{conv} = \dfrac{1}{h\left(\frac{\pi D^2}{4}\right)}$

(ii) Resistance associated with concrete floor $\rightarrow$ $R_{floor} = \dfrac{1}{S' K_{floor}}$

From shape factor tables, $S' = 2D$. $R_{floor} = \dfrac{1}{2K_{floor}D}$

$$R_{ext} = 1\Big/\left(\frac{1}{R_{floor}} + \frac{1}{R_{conv}}\right) = 1\Big/\left(2K_{floor}D + \frac{h\pi D^2}{4}\right)$$

Condition for validity of lumped capacitance model is $Bi < 0.1$

$$\Rightarrow \quad Bi = \frac{R_{int}}{R_{ext}} = \frac{H}{K_{Al}\left(\frac{\pi D^2}{4}\right)}\left(2K_{floor}D + \frac{h\pi D^2}{4}\right) = \boxed{\frac{\frac{8}{\pi}K_{floor}HD + hH}{K_{Al}} < 0.1}$$

There are multiple other ways to solve this problem. One could form a resistance network for the system and use the network to find $Bi = R_{int}/R_{ext}$. Another way would be to individually compare the internal resistance to (i) the convection resistance and (ii) the conduction resistance associated with the floor ($1/2k_{floor}D$) .