University of California, Berkeley
Physics 110B Spring 2004 (*Strovink*)

EXAMINATION 2

Directions: Do all three problems, which have unequal weight. This is a closed-book closed-note exam except for Griffiths, Pedrotti, a copy of anything posted on the course web site, and anything in your own original handwriting (not Xeroxed). Calculators are not needed, but you may use one if you wish. Laptops and palmtops should be turned off. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Show all your work. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Express your answer in terms of the quantities specified in the problem. Box or circle your answer.

Problem 1. (35 points)
A positive point charge q of rest mass m is subjected to a uniform constant electric field E_0 pointing in the $\hat{x}$ direction (as observed in the *laboratory*). This force causes its (relativistic) momentum to increase linearly with laboratory time t.

(**a.**) (15 points)
At $t = 0$ the charge is at rest at the origin. Thereafter, show that $\sinh\eta = \omega_0 t$, where η is the charge's rapidity and ω_0 is a constant. Determine ω_0.

(**b.**) (20 points)
For $t > 0$ show that the charge moves according to

$$\frac{\omega_0}{c}x(t) = \sqrt{1 + \omega_0^2 t^2} - 1 \ .$$

Problem 2. (35 points)
At $t = 0$ at the origin of a laboratory coordinate system, a point particle of charge q has velocity βc directed along the $\hat{z}$ axis. It has been moving with that constant velocity for a long time.

(**a.**) (20 points) At $t = 0$, starting with the Coulomb potential in the particle's rest frame, and using the rules for relativistic transformation both of coordinates and of EM potentials (in Lorentz gauge), find the scalar potential V seen by an observer located at Cartesian coordinates $(x, 0, z)$, where $V(\infty) \equiv 0$. Express V in terms of β, x, z, and constants.

(**b.**) (15 points) As seen at $t = 0$ by the laboratory observer, is $-\nabla V$ expected to point in the radial direction? Explain.

Problem 3. (30 points)
A thin insulating rod, running from $z = -a$ to $z = +a$, carries a static line charge density (Coul/m)

$$\lambda(z) = \lambda_0 \cos\frac{\pi z}{a} \ .$$

Find the *leading term only* in the multipole expansion of the electrostatic potential $V(\vec{r})$. Express your answer in terms of the observer's polar angle θ and the observer's distance r to the origin.