# ME 109 Heat Transfer
# Midterm Exam

**October 28, 2015**
**10:10 - 11:00 am**

Name: . . . . . . . . . . . . . . . . . . . . . . . . . . . .

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Discussion: Tu / Th

### Comments

Write solutions on this exam booklet.

Notes: Permitted 2 pages (each 8.5" x 11", both sides).

Calculator permitted.

3 questions, equal weight.

Reasonable approximations are fine (no penalty for up to 10% error).

Show your work.

TABLE A.1 Thermophysical Properties of Gases at Atmospheric Pressure[a]

| $T$ (K) | $\rho$ (kg/m$^3$) | $c_p$ (kJ/kg · K) | $\mu \cdot 10^7$ (N · s/m$^2$) | $\nu \cdot 10^6$ (m$^2$/s) | $k \cdot 10^3$ (W/m · K) | $\alpha \cdot 10^6$ (m$^2$/s) | $Pr$ |
|---|---|---|---|---|---|---|---|
| Air, $\mathcal{M}$ = 28.97 kg/kmol | | | | | | | |
| 100 | 3.5562 | 1.032 | 71.1 | 2.00 | 9.34 | 2.54 | 0.786 |
| 150 | 2.3364 | 1.012 | 103.4 | 4.426 | 13.8 | 5.84 | 0.758 |
| 200 | 1.7458 | 1.007 | 132.5 | 7.590 | 18.1 | 10.3 | 0.737 |
| 250 | 1.3947 | 1.006 | 159.6 | 11.44 | 22.3 | 15.9 | 0.720 |
| 300 | 1.1614 | 1.007 | 184.6 | 15.89 | 26.3 | 22.5 | 0.707 |

Some Correlations for Flat Plates.

| Correlation | Geometry | Conditions[b] |
|---|---|---|
| $\delta = 5x\, Re_x^{-1/2}$ | Flat plate | Laminar, $T_f$ |
| $C_{f,x} = 0.664 Re_x^{-1/2}$ | Flat plate | Laminar, local, $T_f$ |
| $Nu_x = 0.332 Re_x^{1/2}\, Pr^{1/3}$ | Flat plate | Laminar, local, $T_f$, $Pr \gtrsim 0.6$ |
| $\delta_t = \delta Pr^{-1/3}$ | Flat plate | Laminar, $T_f$ |
| $\overline{C}_{f,x} = 1.328 Re_x^{-1/2}$ | Flat plate | Laminar, average, $T_f$ |
| $\overline{Nu}_x = 0.664 Re_x^{1/2}\, Pr^{1/3}$ | Flat plate | Laminar, average, $T_f$, $Pr \gtrsim 0.6$ |
| $Nu_x = 0.565 Pe_x^{1/2}$ | Flat plate | Laminar, local, $T_f$, $Pr \lesssim 0.05$, $Pe_x \gtrsim 100$ |
| $C_{f,x} = 0.0592 Re_x^{-1/5}$ | Flat plate | Turbulent, local, $T_f$, $Re_x \lesssim 10^8$ |
| $\delta = 0.37x\, Re_x^{-1/5}$ | Flat plate | Turbulent, $T_f$, $Re_x \lesssim 10^8$ |
| $Nu_x = 0.0296 Re_x^{4/5}\, Pr^{1/3}$ | Flat plate | Turbulent, local, $T_f$, $Re_x \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$ |

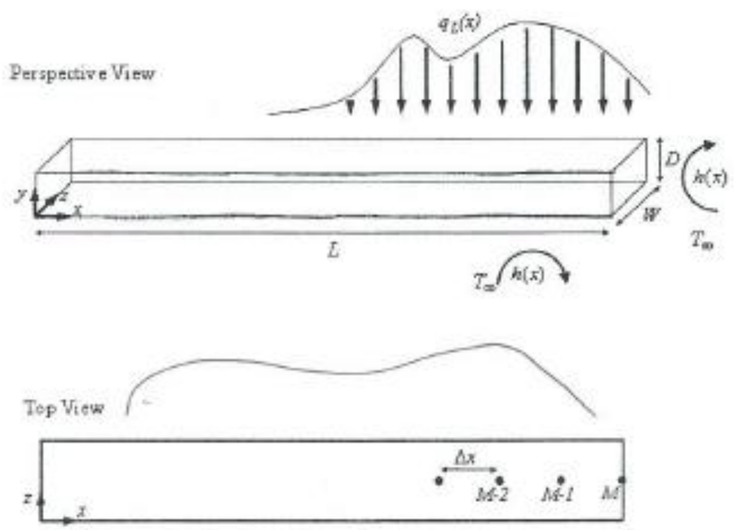
## Problem 1. (20 pts)

Consider a transient finite-difference scheme for T(x,t) of a strip of length L, thickness D, and width W.
The upper surface (y=D) is heated with a laser of specified heat flux $q_L(x)$ [W/m$^2$].
The lower surface (y=0), as well as the surface at (x=L) experience convection to a fluid at constant $T_\infty$ and spatially-varying $h(x)$.
The surfaces at x=0, z=0, and z=W are insulated.
It is appropriate to treat the bar as one-dimensional with uniform spacing between points, $\Delta x$ = const.

*(a: 20 pts) Write down an appropriate finite difference equation for the last node, M.*

Notes:

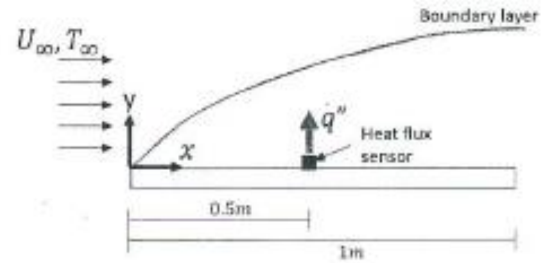
You may use any reasonable scheme (explicit or implicit).
You are *not* required to simplify the final algebraic form.



## Problem 2. (20 pts)

One type of gas-flow velocity meter works by relating the heat transfer rate $q''$ to the free-stream velocity $U_\infty$.

Consider such a velocity meter made of a flat plate of length $L = 1$ m and held at a constant temperature $T_s$=200 K. The flowing gas is air at a temperature $T_\infty = 100$ K. There is a small heat-flux sensor at the location $x_s = L/2 = 0.5$ m.

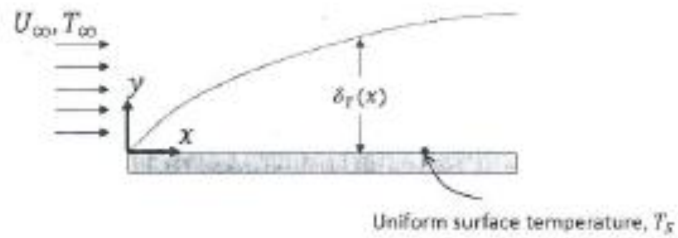



(a: 5 pts) The flow meter is specified by the manufacturer to work for $U_\infty$ in the range ~10 m/s to ~100 m/s. Does this device operate in a laminar or turbulent flow regime? You must justify your answer.

(b: 15 pts). For a certain air flow, the measured heat flux at the small sensor is $q''(x_s) = 20{,}000$ W/m². For this condition, find $U_\infty$. Give a numerical value.

## Problem 3. (20 pts)

A flat plate experiences a novel type of forced convection. As usual, $x$ and $y$ are measured from the leading edge, $T_s$ is the plate surface temperature, and $T_\infty$ is the free-stream fluid temperature, and we know that the hydrodynamic and thermal boundary layers are both reasonably thin. However, the flow field $u(x, y)$ is different than anything you have studied previously, and is *not* given to you. Fortunately, you are provided the following solution for the temperature field,




$$T(x,y)-T_\infty=(T_s-T_\infty)\cdot\exp\left(-\frac{y}{\gamma x^{1/4}}\right)$$

where $\gamma$ is a given constant with units of [(length)$^{3/4}$].

(a: 5 pts) From the given information, obtain a simple expression for the shape of the thermal boundary layer, $\delta_T(x)$. (Define the thermal boundary layer using a convenient "99%" type definition, just like the Blasius convention, i.e., the height at which the local fluid temperature is within 1% of the free-stream value.)

(b: 10 pts) From the given information, obtain a simple expression for the Nusselt number, $\mathrm{Nu}_x$.

(c: 5 pts). Now you are told that $\gamma=10\dfrac{\nu^{1/2}\alpha^{1/4}}{U_\infty^{3/4}}$, where ν is kinematic viscosity and α is thermal diffusivity.

Use this information to express your result from (b) in the functional form $Nu_x=f(\mathrm{Re}_x,\mathrm{Pr})$.