EE 20N, FALL, 2006, MIDTERM 1, AYAZIFAR

MT1.1 (25 Points) Consider a function *f* defined as follows:

$$f : \mathbb{R} \to \mathbb{C}$$
$$\forall t \in \mathbb{R}, \quad f(t) = (-1)^{i|t|}.$$

Each of the following parts can be solved independently of the other.

(a) Determine an expression for, and provide a clear sketch of the graphs of, $|f(t)|$ and $\measuredangle f(t)$, the magnitude and angle, respectively, of function *f*, where $f(t) = |f(t)|e^{i\measuredangle f(t)}$. Be sure to label all the salient features of your graphs.

(b) Let f_e and f_o denote the even and odd components of *f*, respectively, where, $\forall t \in \mathbb{R}$.

$$f(t) = f_e(t) + f_o(t), \quad f_e(t) = \frac{f(t) + f(-t)}{2}, \quad \text{and} \quad f_o(t) = \frac{f(t) - f(-t)}{2}.$$

Determine f_e and f_o. You may do this by showing how each of the components is related to *f*, or providing the graph of each component f_e and f_o.

MT1.2 (30 Points) You can tackle the two parts of this problem independently. Explain your responses succinctly, but clearly and convincingly.

(a) Albert attends the concert *only if* Sally attends the concert. If Blake attends the concert, then Sally does not attend the concert.

Albert is attending the concert. Is Blake attending the concert?

(b) Determine whether the following argument is valid.

Monday, 2 October 2006: If there is no news today of a looming economic depression, nor any revelation of a political scandal in the executive branch, the prime minister will complete the remaining portion of her term in office, *and* the parliament will pass her education overhaul bill into law at the end of the week.

Tuesday, 3 October 2006: The prime minister announced her resignation at 8 am today.

Therefore, her education overhaul bill will never be passed by the parliament.

MT1.3 (25 Points) Consider a function G defined as follows:

$$G : \mathbb{R} \to \mathbb{C}$$
$$\forall \omega \in \mathbb{R}, \quad G(\omega) = \frac{1}{1 + i\,\dfrac{\omega}{\omega_0}},$$

where ω_0 has a fixed positive real value.

Determine an expression for, and provide a clear sketch of the graphs of, the magnitude $|G(\omega)|$ and $\angle G(\omega)$ of function G, where $G(\omega) = |G(\omega)|e^{i\angle G(\omega)}$.
For what value(s) of ω does the function $|G(\omega)|$ attain a maximum? What is the value of $|G(\omega_0)|$? What are the values of $\angle G(\omega)$ for $\omega = -\omega_0, 0, +\omega_0$? Determine the limits:

$$\lim_{\omega \to -\infty} |G(\omega)|, \quad \lim_{\omega \to +\infty} |G(\omega)|, \quad \lim_{\omega \to -\infty} \angle G(\omega), \quad \text{and} \quad \lim_{\omega \to +\infty} \angle G(\omega).$$

You may express your answers to these questions by placing appropriate labels on your sketches.

MT1.4 (25 Points) Consider the discrete-time signal $f : \mathbb{Z} \to \mathbb{R}$, characterized as follows:

$$\forall m \in \mathbb{Z}, \quad f(m) = \begin{cases} 1 & m = 0, 1, 2 \\ 0 & \text{elsewhere.} \end{cases}$$

You can tackle the two parts of this problem independently.

(a) A related signal $p : \mathbb{Z} \to \mathbb{R}$ results from *modulating f*, as follows:

$$\forall m \in \mathbb{Z}, \quad p(m) = \frac{1}{2}\left[1 + (-1)^m\right] f(m).$$

Provide a well-labeled sketch of the signal *p*.

(b) A related signal $q : \mathbb{Z} \to \mathbb{R}$ results from the *convolution* of the signal *f* with itself; this is written as $q = f * f$, or $q(n) = (f * f)(n)$, $\forall n \in \mathbb{Z}$. In particular, the signal *q* satisfies the following *convolution sum*:

$$\forall n \in \mathbb{Z}, \quad q(n) = \sum_{m=-\infty}^{\infty} f(m)\, f(n-m).$$

Provide a well-labeled sketch of graph(*q*). (This would be a stem plot, that is, a "lollypop" plot.)

Hint: Discrete-time convolution is generally simpler than continuous-time convolution. Start by sketching the signal *f* as a function of *m*. Also, plot the "time-reversed and shifted" version of *f* (i.e., *f(n-m)*) as a function of *m*, for various values of *n*. Then perform point-wise multiplication and summation, as suggested by the convolution sum above (but do it graphically!). Try to determine for what values of *n* the convolution sum is zero, so you know what values of *n* to focus your attention on.