Math 54. Solutions to First Midterm

1. (8 points) Suppose A is a 5×3 matrix and $\vec{b}$ is a vector in $\mathbb{R}^5$ with the property that $A\vec{x} = \vec{b}$ has a unique solution. What can you say about the reduced echelon form of A? Justify your answer.

2. (12 points) Express the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 8 & 5 \end{bmatrix}$$

as a product of elementary matrices.

3. (10 points) Compute the determinant

$$\begin{vmatrix} 2 & 0 & 10 & 11 & 8 & 9 & 0 \\ 0 & 3 & 11 & 13 & 10 & 5 & 0 \\ 0 & 0 & 1 & 2 & 1 & 3 & 0 \\ 0 & 0 & 1 & 3 & 2 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 3 & 2 & 0 \\ 0 & 0 & 9 & 4 & 8 & 7 & 2 \end{vmatrix} .$$

You may lose points if you need more computation than is necessary.

4. (10 points) Let

$$W = \{\vec{p} \in \mathbb{P}_3 : \vec{p}(1) = \vec{p}\,'(2) + \vec{p}\,''(3)\} .$$

Is W a subspace of $\mathbb{P}_3$? Explain.

5. (10 points) Use coordinate vectors to test whether the following set of polynomials spans $\mathbb{P}_2$. Justify your conclusion.

$$1 - t + 2t^2\,,\ 2 + 5t^2\,,\ t + t^2\,,\ 3 - 3t + 8t^2\,.$$