EE 20N: Structure and Interpretation of Signals and Systems
Department of Electrical Engineering and Computer Sciences — MIDTERM 3
UC BERKELEY — 30 April 2015

FIRST Name Solutions LAST Name ____________

Lab Day & Time: ________ SID (All Digits): ________

Name of the person to your left: ____________________

Name of the person to your right: ____________________

- **(10 Points)** Print your *official* name (not your e-mail address) and *all* digits of your student ID number legibly, and indicate your lab time, on *every* page.
- This exam should take up to 80 minutes to complete. You will be given at least 80 minutes, up to a maximum of 90 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except three double-sided 8.5" × 11" sheets of handwritten, original notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 7.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the seven numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

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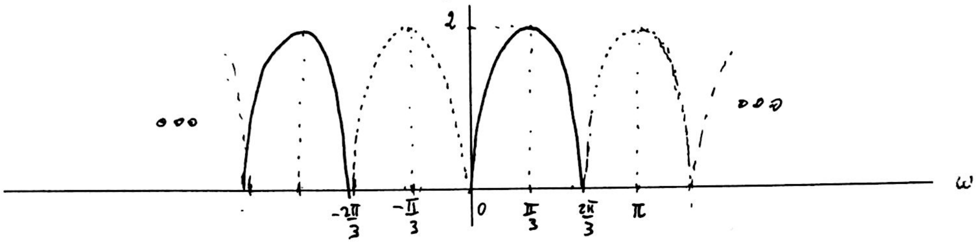
Lab Day & Time: ________ SID (All Digits): ________

MT3.1 (15 Points) Sketch the magnitude and angle of the frequency response of an LTI system with the unit-impulse response

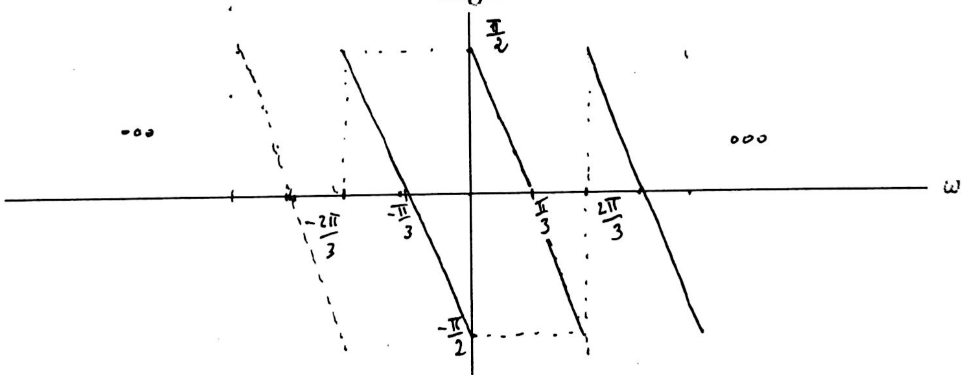
$$h[n] = \delta[n] - \delta[n-3]$$

Label all important magnitudes, angles, and frequencies. Axis scales should be linear.

magnitude



angle



antisymmetric (or odd symetry)

phase: $\omega \in (0, \frac{2}{3}\pi) \rightarrow \sin(\frac{3}{2}\omega) > 0$

so $\angle H(\omega) = \frac{\pi}{2} - \frac{3}{2}\omega + 0$

$@\ \omega = 0 \longrightarrow \angle H(0) = \frac{\pi}{2}$

$@\ \omega = \frac{2}{3}\pi \longrightarrow \angle H(\frac{2\pi}{3}) = \frac{\pi}{2} - \pi = -\frac{\pi}{2}$

$@\ \omega = \frac{\pi}{3} \rightarrow \angle H(\frac{\pi}{3}) = 0$

$h[n] = \delta[n] - \delta[n-3]$

$H(\omega) = 1 - e^{-3j\omega}$

$= 2j e^{-\frac{3}{2}j\omega} \cdot \left(\frac{e^{\frac{3}{2}j\omega} - e^{-\frac{3}{2}j\omega}}{2j} \right)$

$= 2j e^{-\frac{3}{2}j\omega} \sin(\frac{3}{2}\omega) \longrightarrow \frac{2}{3} \cdot 2\pi = \frac{4}{3}\pi$ periodic

magnitude: $|H(\omega)| = 2\left|\sin(\frac{3}{2}\omega)\right| \longrightarrow$ even symmetry $\rightarrow$ enough to plot interval $\frac{(\frac{4}{3}\pi)}{2} = \boxed{\frac{2}{3}\pi}$

$\angle$

at $\omega = 0 \longrightarrow |H(0)| = 0$

at $\omega = \frac{1}{3}\pi \rightarrow |H(\frac{\pi}{3})| = 2$

at $\omega = \frac{2}{3}\pi \rightarrow |H(\frac{2\pi}{3})| = 0$

FIRST Name ______________________ LAST Name ______________________

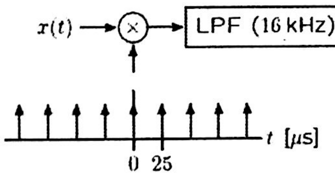
Lab Day & Time: ________ SID (All Digits): ________

MT3.2 (20 Points) Let $x(t)$ be a periodic signal with the following harmonics:

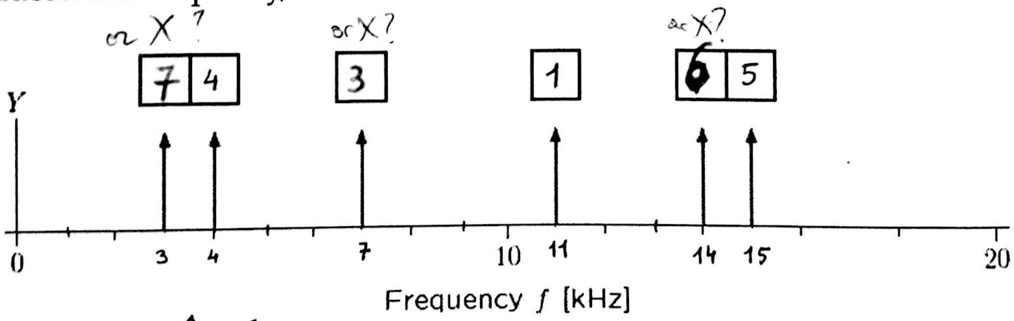
harmonic number	frequency (kHz)
1	11
2	22
3	33
4	44
5	55
6	66
7	77

Throughout this problem, frequencies (f) are expressed in cycles per second (Hz), which are related to corresponding angular frequencies Ω (rad/s) by $f = \frac{\Omega}{2\pi}$.

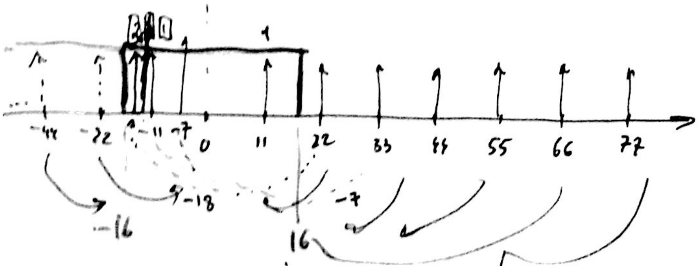
The signal $x(t)$ is multiplied by an infinite train of impulses separated by 25×10^{-6} s, and the result is passed through an ideal low-pass filter with a cutoff frequency of 16 kHz.

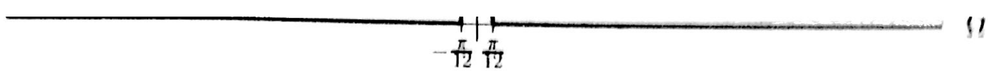


The plot below shows the Fourier transform Y of the output signal, for frequencies between 0 and 20 kHz. Write the number of the harmonic of $x(t)$ that produced each component of Y in the box above that component. If none of 1–7 could have produced this frequency, write X.



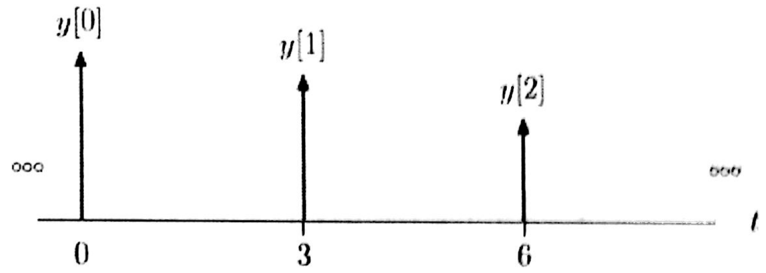
method 1





We wish to compare two methods of using $y[n]$ to reconstruct approximations to $x(t)$.

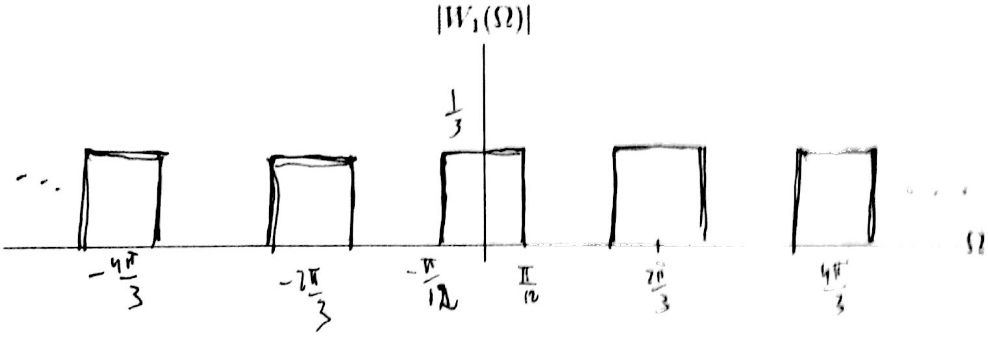
(a) (15 Points) Let $w_1(t)$ represent a signal in which each sample of $y[n]$ is replaced by an impulse of area $y[n]$ located at $t = 3n$. Thus $w_1(t)$ has the following form:



which can be represented mathematically as

$$w_1(t) = \sum_{n=-\infty}^{\infty} y[n]\delta(t-3n)$$

Sketch the magnitude of the Fourier transform of $w_1(t)$ on the axes below. *Label all important features.*



$$w_1(t) = \sum_{n=-\infty}^{+\infty} y[n]\delta(t-3n)$$

$$= \sum_{n=-\infty}^{+\infty} x(3n)\delta(t-3n) = x(t)\cdot\sum_{n=-\infty}^{+\infty}\delta(t-3n)$$ because of sampling property of $\delta(\ )$

$\downarrow$

$$W_1(\Omega) = \frac{1}{2\pi}X(\Omega) * \frac{2\pi}{3}\sum_{k=-\infty}^{+\infty}\delta\left(\Omega-\frac{2\pi}{3}k\right) = \frac{1}{3}\frac{\cancel{2\pi}}{\cancel{2\pi}}\sum_{k=-\infty}^{+\infty}X\left(\Omega-\frac{2\pi}{3}k\right)$$

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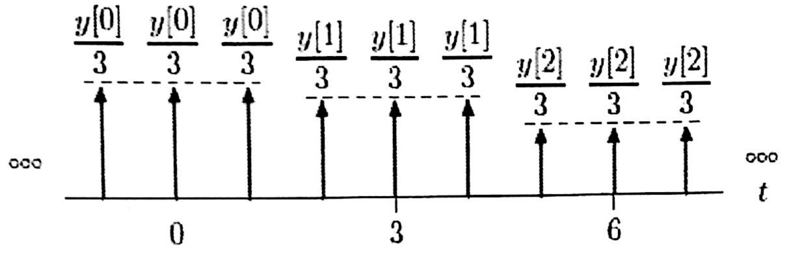
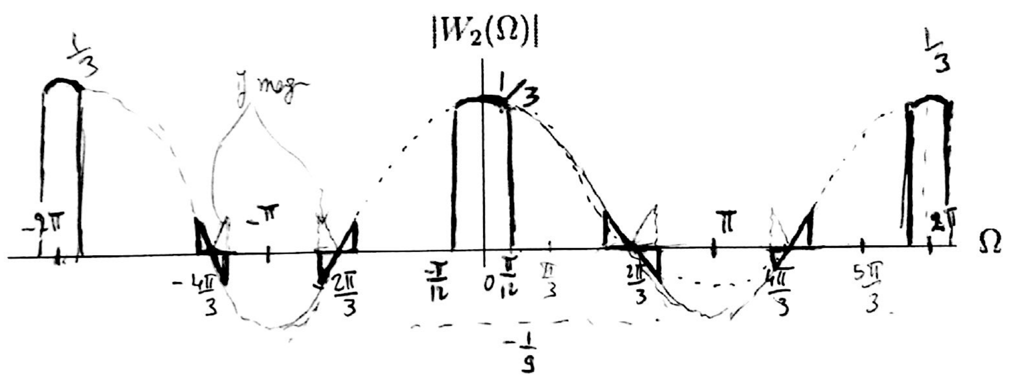
(b) (15 Points) Let $w_2(t)$ represent a signal in which each sample of $y[n]$ is replaced by three impulses, one at $t = 3n - 1$, one at $t = 3n$, and one at $t = 3n + 1$, each with area $y[n]/3$. Thus $w_2(t)$ has the following form:



which can be represented mathematically as

$$w_2(t) = \tfrac{1}{3} \sum_{n=-\infty}^{\infty} y[n] \left(\delta(t - 3n - 1) + \delta(t - 3n) + \delta(t - 3n + 1)\right)$$

Sketch the magnitude of the Fourier transform of $w_2(t)$ on the axes below. *Label all important features, including the value of* $|W_2(\Omega)|$ *at* $\Omega = 0$.



$w_2(t) = \left[w_1(t) + w_1(t-1) + w_1(t+1)\right] \frac{1}{3}$.

time delay property

$W_2(\Omega) = \left[W_1(\Omega) + e^{-j\omega\Omega} W_1(\Omega) + e^{j\Omega} W_1(\Omega)\right] \frac{1}{3}$

$= \left(W_1(\Omega) \left(1 + 2\cos(\Omega)\right)\right) \frac{1}{3}$.

@ $\Omega = \frac{2\pi}{3} \longrightarrow W_2 = 0$

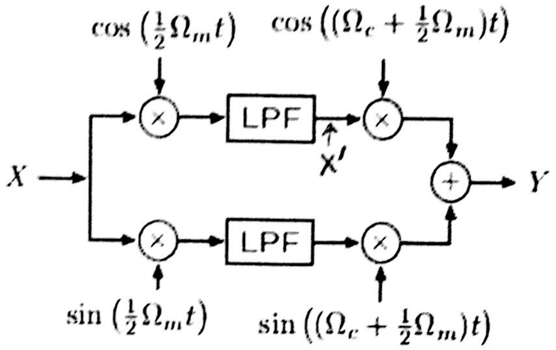
$\Omega = \frac{4\pi}{3} \longrightarrow W_2 = 0$

$\Omega = \frac{6\pi}{3} \longrightarrow W_2 = \frac{1}{3}$

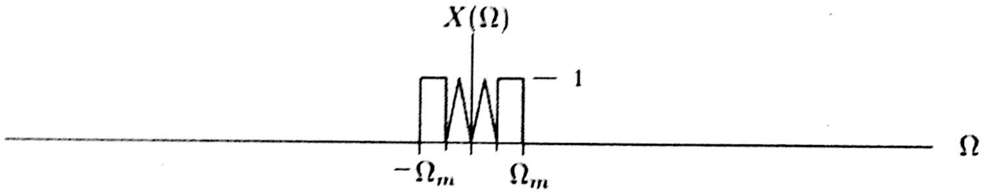
FIRST Name ______________________ LAST Name ______________________

Lab Day & Time: __________ SID (All Digits): __________

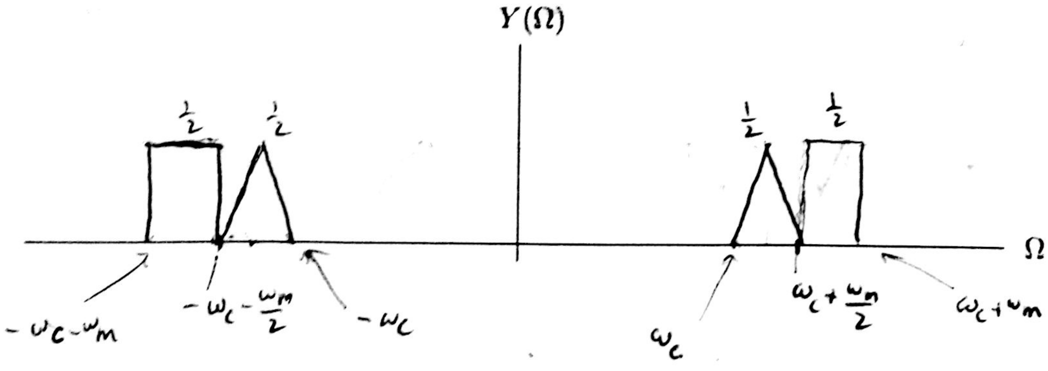
MT3.4 (40 Points) Consider the following modulation scheme, where $\Omega_c \gg \Omega_m$.



Assume that each low-pass filter is ideal, with cutoff frequency $\frac{1}{2}\Omega_m$. Also assume that the input signal has the following Fourier transform $X(\Omega)$:



Sketch $Y(\Omega)$ on the following axes. *Label all important magnitudes and frequencies.*



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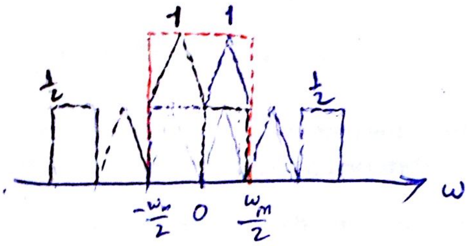
(This is a blank page for your scratch work on this problem. Your final answer must go on the axes on the previous page, but we will use this page to assess partial credit, so make sure it is clear what you are doing.)

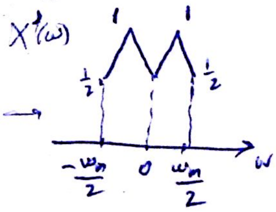
upper route (1)

operation 1.1 modulation

$x(t)\cdot\cos(\frac{1}{2}\omega_m t) \xrightarrow{\mathcal{F}} \frac{1}{2}\left[X(\omega-\frac{\omega_m}{2})+X(\omega+\frac{\omega}{2})\right]$



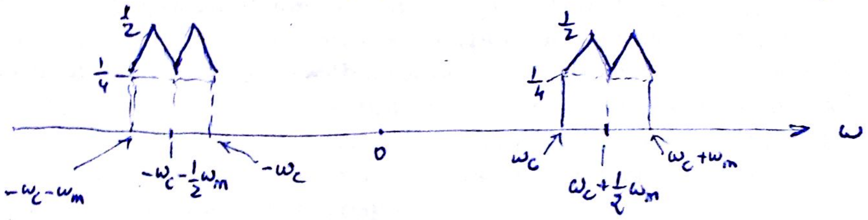
operation 1.2 - LPF →



operation 1.3 modulation with carrier freq. $\omega_c >> \omega_m$

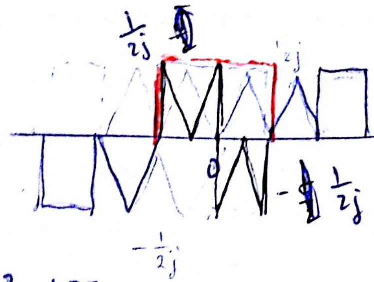
$x'(t)\cdot\cos((\omega_c+\frac{1}{2}\omega_m)t) \xrightarrow{\mathcal{F}} \frac{1}{2}\left[X'(\omega-(\omega_c+\frac{1}{2}\omega_m))+X'(\omega+(\omega_c+\frac{1}{2}\omega_m))\right]$

X →

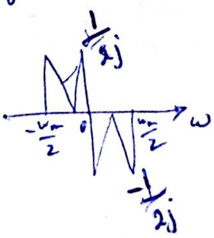


lower route (2) operation 2.1: modulation with sin

$x(t)\sin(\frac{1}{2}\omega_m t) \xrightarrow{F} \frac{1}{2j}\left[X(\omega-\frac{\omega_m}{2})-X(\omega+\frac{\omega_m}{2})\right]$



operation 2.2 - LPF →



operation 2.3. mod with carrier $\omega_c+\frac{1}{2}\omega_m$