Berkeley Physics H7B Fall 2015
Dr. Winoto - Midterm 1 Examination
Wednesday, September 30th, 2015

Instruction for the examination (please read carefully):
- Topic: Thermodynamics
- There are 4 problems total (NOT in any order of length and difficulty, they vary, but all of them are worth the same), do them in any order you prefer.
- Total points for the exam = 100 points for a perfect score
- You have exactly 125 minutes to complete the test
- Please outline and explain in details all your physical and mathematical reasonings in a clear, concise, step-by-step and logical manner.

1. (25 points): Carnot Engine:
An infinite heat reservoir M1 is at a constant temperature T_H. A finite system or reservoir M2 with a constant heat capacity C_2, is initially at temperatures T_2. T_H is greater than T_2.
(a). Suppose we bring M2 into thermal contact with M1. Eventually, they reach a thermal equilibrium. Please calculate the entropy change of M2, the entropy change of M1, and the total entropy change. (ONLY if you have time, show explicitly that the total entropy change is strictly greater than zero).
Now, suppose INSTEAD of (a):
we use M1 and M2 as a high temperature reservoir and a low temperature reservoir, respectively, to run a Carnot engine. Or in other words, we use the Carnot engine to transfer heat from M1 into M2 until they reach a thermal equilibrium, after an infinite number of infinitesimal Carnot cycles and the engine comes to rest eventually.
(b). (12pts) Calculate the total final amount of work W done by the Carnot engine.
(c). Calculate explicitly: the entropy change for M1, the entropy change for M2, and the entropy change for the whole system (M1 + M2 + Carnot engine). And justify your answer for the entropy change for the whole system.

2. (25 points): Isothermal Atmosphere:
Consider a vertical, very long and closed column of mono-atomic ideal gas, with one end of the column at ground level of the surface of the earth. The number of gas particle is N, and the mass of each particle is m. The whole column is to be considered at thermal equilibrium at a constant and homogeneous temperature T. The cross-section area of the column is A, and the length is L, where L is very large, approaching ∞, but at the same time we will also assume that the gravitational acceleration g is constant through out the column (downward, and equal to 9.8 m/s^2 but you won't need this number). And in case, you don't remember from Physics 7A, the potential energy of a particle at height z in this constant gravitational field is given by mgz.
(a). Using the equipartition theorem, please calculate the average kinetic energy of the gas.
(b). (10pts) Using the Boltzmann distribution, calculate the average potential energy of the gas.
(c). Calculate the heat capacity at constant volume C_V for the gas.
(d). Please calculate the pressure of the gas at $z=\infty$ and at $z=0$.

3. (25 points): Adiabatic Process (see Figure #3):
A mono-atomic ideal gas of N particles is at a state A with a pressure of p_o, volume V_o, and temperature T_o. The gas undergoes an adiabatic expansion to a volume of $2V_o$(state B).
(a). Please calculate the temperature of the gas at state B, T_B.
(b). Calculate the entropy change $\Delta S(A \text{ -> } B)$ for this process.

Now, suppose instead of the adiabatic expansion, the process, from *A* to *B*, follows the following path-way (as shown in the figure):
- first, an isovolumetric reduction in temperature from T_o to T_B (*A* -> *C*)
- second, an isothermal expansion at temperature T_B from V_o to $2V_o$ (*C* -> *B*)

(c). Calculate the heat transfers into the gas from *A* -> *C* and also from *C* -> *B*.
(d). (10pts) Calculate the entropy change of the gas from *A* -> *C,* the entropy change from *C* -> *B*, and also the total entropy change from *A* -> *C* -> *B*. Since entropy is a state function, please show explicitly that your answer for ΔS(*A* -> *C* -> *B*) is equal to the answer from part (b).

4. (25 points): Simple Harmonic Oscillation:
Consider a 1D (along the *x*-axis) mono-atomic ideal gas of *N* particles of mass *m* in a simple harmonic potential energy given by $\frac{1}{2}k_s x^2$, where $-\infty < x < \infty$ and k_s is the spring constant. The ideal gas is at a contant temperature *T*.
(a). Using the equipartition theorem, please calculate the average kinetic energy of the gas.
(b). (10pts) Using the Boltzmann distribution, calculate the average potential energy of the gas.
(c). (10pts) Calculate the heat capacity at constant volume C_V for the gas.

$$\int_{-\infty}^{\infty} x^2 e^{-\alpha x^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}.$$

This trick can be repeated with equal ease. Differentiat
gives

$$\int_{-\infty}^{\infty} x^4 e^{-\alpha x^2}\, dx = \frac{3}{4}\sqrt{\frac{\pi}{\alpha^5}}.$$

Therefore we have a way of generating the integrals betw
and ∞ of $x^{2n}e^{-\alpha x^2}$, where $n \geq 0$ is an integer.[1] Becau
functions are even, the integrals of the same functions b
and ∞ are just *half* of these results:

$$\int_{0}^{\infty} e^{-\alpha x^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}},$$

$$\int_{0}^{\infty} x^2 e^{-\alpha x^2}\, dx = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}},$$

$$\int_{0}^{\infty} x^4 e^{-\alpha x^2}\, dx = \frac{3}{8}\sqrt{\frac{\pi}{\alpha^5}}.$$

$$n! = \int_0^{\infty} x^n e^{-x}\, dx.$$