Berkeley Physics H7B Spring 2015
Dr. Winoto - Midterm 2 Examination
Thursday, April 9th, 2015

Instruction for the examination (please read carefully):
- Topic: Purcell Ch. 1-5.
- There are 3 problems, **do them in any order you prefer**.
- Total points for the exam = 100 points for a perfect score
- You have exactly (90-10=80) minutes to complete the test
- Please explain in details all your physical and mathematical reasonings in a clear, step-by-step and logical manner.

1. (30 points): Relativistic capacitors (see Figure 1 on the board)**:**
Carrying 2 identical square parallel plate capacitors with length L and separation s ($s \ll L$, so that we can safely ignore any fringe effect), Ivan is cruising relativistically past Yara (who is stationary in the lab frame) in the $+x$-direction with speed v. On his left hand is one of the capacitors (C_L) oriented such that the plane of the capacitor plate is parallel with respect to the xy-plane, as shown in Figure 1. On his right hand is the other capacitor (C_R), oriented such that the plane of the capacitor plate is parallel with respect to the yz plane. Both capacitors have charges +Q and -Q on each of their plates.
(a). By integrating the electrostatic energy density inside each capacitor, please calculate the electrostatic energy of each capacitor according to Ivan.
(b). Please calculate the electric-field inside each capacitor (C_L and C_R) according to Yara.
(c). Also, by integrating the electrostatic energy density inside each capacitor, please calculate the electrostatic energy of each capacitor (C_L and C_R) according to Yara.
(d). (0 point) If you have time left, please give some thought to (c). Are you surprised by the answers you get or is it as you expected? Since E=mc^2, if Yara somehow can put these two moving capacitors on a scale, will the balance tip or not tip? If the balance tips, then, is there anything missing?, for how could the energy of the capacitor depend on its orientation!? If the balance doesn't tip, then everything is fine, no? Don't let your relativistic doubt or certainty on part(d), amusing sure but irrelevant to the problem at hand, to influence your answer to the relatively simple problem of part (c) :).

2. (35 points): Electric-dipole and Induced electric-dipole: (see Figure 2):
Consider a permanent electric dipole $+q$ and $-q$, separated by a in the z direction, centered at the origin as shown in Figure 2; The dipole moment is $p_o = qa$, pointing in the +z direction. As you know, the E-field of the permanent dipole (for $r \gg a$) is given by the following expression:

$$\vec{E}(\mathrm{r},\theta) = \frac{p_o}{4\pi\epsilon_o}\frac{1}{r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$, where θ is the angle from the z-axis.

Now, suppose one put a neutral atom at a distance x far from the origin along the $+x$-axis, as shown in Figure 2. As a toy model, one can think of a neutral atom as an electric dipole $+e$ and $-e$ connected by a spring with a spring constant k_s, and when the atom is unperturbed, the internal Coulomb force between $+e$ and $-e$ is balanced by the spring at a spring separation of Δ=0, i.e. the $+e$ and $-e$ are on top of each other (don't ask why! but it is a pretty good toy model for what is in reality a quantum mechanical problem).
(a). Please calculate the E-field vector of the permanent dipole **at** the position of the atom.
(b). Please calculate the new separation Δ (magnitude and direction) of the electric dipole of the atom.
(c). What is the induced dipole moment p_i of the atom (magnitude and direction) due to the E-field of the permanent dipole p_o.
(d). Calculate the E-field due to this induced dipole p_i **at** the position of the permanent dipole. What is

the distance dependence of this induced E-field?

(e). (Extra credit 5pts) Please give a qualitative answer to the magnitude and the direction of the induced force **on** the permanent dipole due to the atom. Also, elaborate on whether this force is attractive or repulsive.

3. (35 points): Potential and charge distribution:

A spherically symmetric charge distribution gives rise to a spherically symmetric electrostatic potential given by the following:

$$V(r) = q\,\frac{\mathrm{Exp}(-\alpha r)}{r}\ ;\ \alpha > 0.$$

where r is the radial distance from the origin.

(a). Evaluate the E-field (magnitude and direction) everywhere as a function of *r*.

(b). Use Gauss' Law to calculate the amount of charge inside a sphere of radius *r*.

(c). Is there any point charge present anywhere? If so, where? And, what is its magnitude?

(d). Either use Poisson' equation in spherical coordinate or use your answer to part (a,b,&c) to calculate the volume charge density $\rho(r)$ as a function of *r*.

Acknowledgement:

I would like to thank Yara and Ivan, two very brave students in our class, who become the first ever astronouts of relativistic thought experiments, both in class and in this examination.