Problem 1

There are many ways to approach and do this problem. I'm going to present what I think is the most straightforward way here.

We want to ultimately find a velocity, and the easiest way to do this is through conservation of energy, which means we need to find the work done by the field, which means we just need to find the *potential difference* between the two points. Since we have a continuous charge distribution, we need to do an integral to find ϕ, the potential [some people use V, which is just a matter of preference.] The template integral for potentials is

$$\Delta\phi = \int \frac{dq}{4\pi\epsilon_0 r}$$

So, we need to find dq, r, and appropriate limits. Let's say that the charge is a distance D away from the line of charge, and place our origin of coordinates at the right end of the line and integrate to the left [there are many other choices of origins and directions of integration that are all equivalent]. Then, the point charge is at a distance D to the right of the origin, and the integration variable will be x, where x gives the distance along the line from the right edge of the line of charge. Thus, $r = x + D$ and we are integrating from $x = 0$ to $x = L$. This is a line charge of uniform linear charge density λ, so $dq = \lambda dx$. Substituting in gives

$$\begin{aligned}
\phi(D) &= \int_0^L \frac{\lambda dx}{4\pi\epsilon_0(x+D)} \\
&= \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{dx}{x+D} \\
&= \frac{\lambda}{4\pi\epsilon_0}\int_D^{L+D} \frac{du}{u} \\
&= \frac{\lambda}{4\pi\epsilon_0}\ln(u)\Big|_D^{L+D} \\
&= \frac{\lambda}{4\pi\epsilon_0}\ln\Big(\frac{L+D}{D}\Big)
\end{aligned}$$

Now, we have the charge starting from a distance $D = d$ and ending at $D = 2d$, so the potential difference is

$$\begin{aligned}
\Delta\phi(D) &= \phi(2d) = \phi(d) \\
&= \frac{\lambda}{4\pi\epsilon_0}\Big[\ln\Big(\frac{L+2d}{2d}\Big) - \ln\Big(\frac{L+d}{d}\Big)\Big] \\
&= \frac{\lambda}{4\pi\epsilon_0}\ln\Big(\frac{L+2d}{2d}\frac{d}{L+d}\Big) \\
&= \frac{\lambda}{4\pi\epsilon_0}\ln\Big(\frac{L+2d}{2L+2d}\Big)
\end{aligned}$$

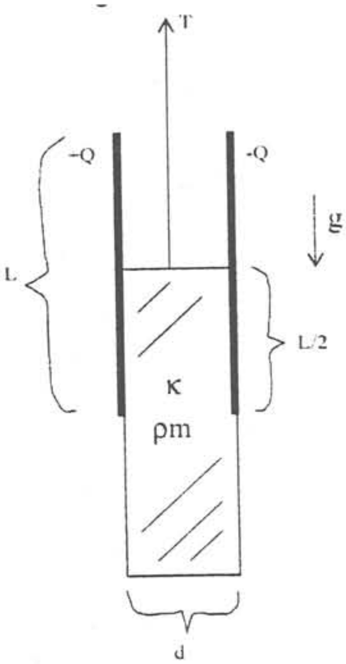
We can now invoke conservation of energy, which tells us

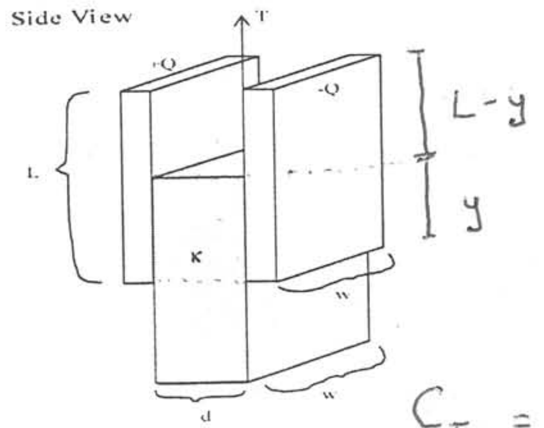
$$\begin{aligned}
U_i + K_i &= U_f + K_f \\
U_i - U_f &= {}^1\!/_2 mv^2 \\
v^2 &= \frac{-2Q\Delta\phi}{m}
\end{aligned}$$

It is important to note the negative sign. If we forgot that, then we would have v^2 wind up negative giving an imaginary velocity! So, all we have to do is plug in, and use the minus sign to flip the fraction within the logarithm to get

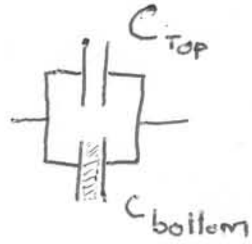
$$v = \sqrt{\frac{\lambda Q}{2\pi\epsilon_0 m} \ln\left(\frac{2d + 2L}{2d + L}\right)}$$

Some physical checks: The argument of the logarithm is positive, greater than one, and unitless, giving a positive, unitless number. The other parameters under the square root are all positive [λQ are both the same sign, otherwise the charge would not be pushed away from the line charge!], so our velocity is real. For units, we have a charge squared over an ϵ_0. Comparing with Coulombs law shows that the units of this are $kg \cdot m/s^2 \cdot m^2$, so the argument of the square root has dimensions $kg \cdot m^3 \cdot s^{-2} \cdot m^{-1} \cdot kg^{-1} = (m/s)^2$, so our answer is in units of m/s!





$\Rightarrow$



Capacitors in Parallel

$$C_{Top} = \frac{\epsilon_0 (L-y) w}{d}$$

$$C_{Bottom} = \frac{\kappa \epsilon_0 y w}{d}$$

$$C_{effective} = C_{Top} + C_{bottom} = \frac{\epsilon_0 (L-y) w}{d} + \frac{\kappa \epsilon_0 y w}{d}$$

$$\Rightarrow C_{effective} = \frac{\epsilon_0 w}{d}\left(L + y(\kappa - 1)\right)$$

$$\Rightarrow U = \frac{1}{2}\frac{Q^2}{C_{eff}} = \frac{Q^2 d}{2\epsilon_0 w\left(L + y(\kappa-1)\right)}$$

FBD

$$\sum F_y = m \overset{0 \text{ Static}}{\cancel{a_y}}$$

$$\overset{0 \text{ given}}{\cancel{T}} + F_c - F_m = 0$$

$$F_c = F_m$$

$$F_{mass} = \rho g w L d$$

$$F_{capacitor} = F_{Mass}$$

$$F_{capacitor} = \left|-\frac{dU}{dy}\right|_{\text{at } y = L/2}$$

$$= \left.\frac{Q^2 d(\kappa-1)}{2\epsilon_0 w\left(L + y(\kappa-1)\right)^2}\right|_{y=L/2}$$

$$= \frac{Q^2 d(\kappa-1)}{2\epsilon_0 w\left(L + \frac{L}{2}(\kappa-1)\right)^2}$$

$$F_{capacitor} = \frac{2Q^2 d}{\epsilon_0 w L^2}\,\frac{\kappa-1}{(1+\kappa)^2}$$

$$\frac{2Q^2 \cancel{d}}{\epsilon_0 w L^2}\,\frac{\kappa-1}{(1+\kappa)^2} = \rho g w L \cancel{d}$$

$$Q^2 = \frac{\rho g w^2 L^3 \epsilon_0}{2}\,\frac{(1+\kappa)^2}{(\kappa-1)}$$

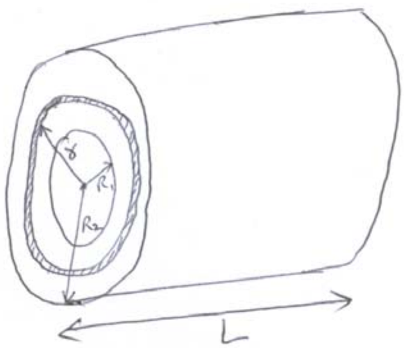
$$\boxed{Q = w(1+\kappa)\sqrt{\frac{\epsilon_0 \rho g L^3}{2(\kappa-1)}}}$$

Problem # 5 Solution

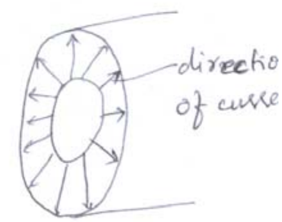
Professor Packard

Mid Term 2

Formula, $R = \rho \dfrac{l}{A}$



The current flows in the radial direction.



$$\therefore \left.\begin{aligned} l &= dr \\ A &= 2\pi r L \end{aligned}\right\} \text{ for the segment shown}$$

$$\therefore \quad dR = \rho \frac{dr}{2\pi r L}$$

$$R = \int_{R_1}^{R_2} dR = \rho \int_{R_1}^{R_2} \frac{dr}{2\pi r L}$$

$$\boxed{R = \frac{\rho}{2\pi L} \ln\left(\frac{R_2}{R_1}\right)}$$

Problem 6 solution: (Packard Midterm 2, Fall 04)

A voltmeter is a galvanometer in series with a resistor with high resistance, R_{ser} (Ref. page 674 of text book)

Since the full scale current sensitivity is 50 μA and the full scale for the voltmeter is 2 volts, from Ohm's law, V=IR, we have

$$2V = (50\mu A)(r + R_{ser})$$

So

$$R_{ser} = \frac{2V}{50\mu A} - 30\Omega = 39970\Omega$$

Further, we have

$$R_{total} = R + \frac{1}{\frac{1}{R} + \frac{1}{r + R_{ser}}} = 1\times10^6\,\Omega + \frac{1}{\frac{1}{1\times10^6\,\Omega} + \frac{1}{40000\Omega}} = 38461.5\Omega$$

Then, I_1 (current passed the left resister) equals to

$$I_1 = \frac{\varepsilon}{R_{total}} = \frac{3V}{38461.5\Omega} \approx 2.89\mu A$$

According to Kirchhoff's loop rule, we know

$$\begin{aligned}\varepsilon &= I_1 R + I_2 (r + R_{ser}) \\ &= I_1 R + I_3 R\end{aligned}$$

According to Kirchhoff's junction rule, we have

$$I_1 = I_2 + I_3$$

We can solve the equations to get

$$I_2 = \frac{\varepsilon - I_1 R}{r + R_{ser}} = 2.75\mu A$$

For the voltmeter, we have

$$\frac{I_2}{I_{full}} = \frac{V_{read}}{V_{full}}$$

Therefore

$$V_{read} = \frac{I_2}{I_{full}} V_{full} = \frac{2.75\mu A}{50\mu A}(2V) = 0.11V$$