Fall 2015 Midterm-2

1. Answer the following questions concisely. Use the semiconductor parameters in the following Table if needed.

Electron diffusion coefficient	D_n	30 cm^2/s
Hole diffusion coefficient	D_p	10 cm^2/s
Electron mobility	μ_n	1200 cm^2/s-V
Hole diffusion coefficient	μ_p	400 cm^2/s-V
Electron diffusion length	L_n	2 μm
Hole diffusion length	L_p	1 μm

a. Two NPN BJT's have identical doping levels. The base width of BJT-A is half that of BJT-B. Under the same base-emitter bias voltage, which BJT has higher collector current?

b. What is the **ratio** of the collector currents in NPN and PNP BJT's if they have identical dimensions and doping concentrations (but opposite doping types)?

c. What is the **ratio** of the transconductances (g_m) of NMOS and PMOS if they have identical dimensions, doping concentrations, magnitudes of threshold voltages and gate-to-source bias voltages?

(a) $I_c = I_s e^{V_{BE}/V_T}$, $I_s = \frac{qA_E D_n n_i^2}{N_A W} \propto \frac{1}{W}$

So BJT-A will have 2x higher collector current,

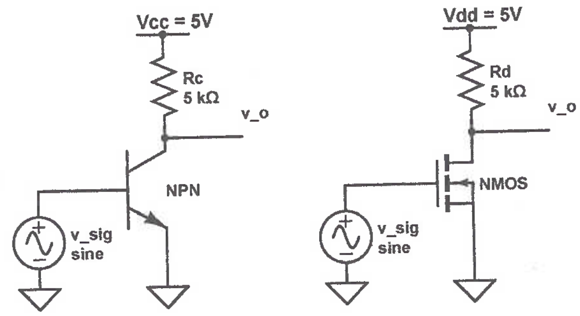
(b) $\frac{I_c(NPN)}{I_c(PNP)} = \frac{D_n}{D_p} = \frac{30}{10} = 3$

(c) $I_D(NMOS) = \frac{k_n}{2}(V_{GS} - V_{tn})^2$, $g_m(NMOS) = k_n(V_{GS} - V_{tn})$

$I_D(PMOS) = \frac{k_p}{2}(|V_{GS}| - |V_{tp}|)^2$, $g_m(PMOS) = k_p(|V_{GS}| - |V_{tp}|)$

$\frac{g_m(NMOS)}{g_m(PMOS)} = \frac{k_n}{k_p} = \frac{\mu_n C_{ox}}{\mu_p C_{ox}} = \frac{1200}{400} = 3$

2. Consider the two amplifiers shown below (only the AC circuit is shown). The NPN BJT has a current gain of 100, and a $v_{CE,sat} = 0.3V$, and the NMOS has $k_n = 1\ mA/V^2$ and $V_{tn} = 1V$. Ignore Early effects. Assume both transistors are biased at 0.5 mA.



   a. Find the voltage gains of both amplifiers. Which amplifier has higher gain?
   b. Which amplifier (BJT or NMOS) has higher input resistance? What's its value?
   c. What are the output resistances of both amplifiers?
   d. Which amplifier (BJT or NMOS) has large output swing? What's its value? (*Note: output swing is defined as the smaller of the upward and downward voltage swings)*

$I_C = I_D = 0.5\ mA$

BJT. $g_m = \frac{I_C}{V_T} = 40 \times 0.5 = 20\ mS$

MOS. $I_D = \frac{1}{2} k_n V_{ov}^2 = 0.5\ mA$. $k_n = 1\ mA/V^2$

$\Rightarrow V_{ov} = 1V$

$g_m = k_n V_{ov} = 1\ mS$

(a) $A_v(BJT) = -g_m R_C = -100\ V/V$

$A_v(MOS) = -g_m R_D = -5\ V/V$

(b) $R_{in}(BJT) = r_\pi = \frac{\beta}{g_m} = 100 \times 50 = 5k\Omega$

$R_{in}(MOS) = \infty \Rightarrow$ larger

(c) $R_o(BJT) = R_C = 5k\Omega$

$R_o(MOS) = R_D = 5k\Omega$

(d) BJT. $V_C = 5 - I_C R_C = 2.5V$

Upward swing $v(+) = V_{CC} - V_C = 2.5V$

downward " $v(-) = V_C - V_{CE,sat} = 2.5 - 0.3 = 2.2V$

Output swing $= \pm 2.2V$

(Continuation of Problem 2 solution)

MOS : $V_D = V_{DD} - I_D R_D = 2.5V$

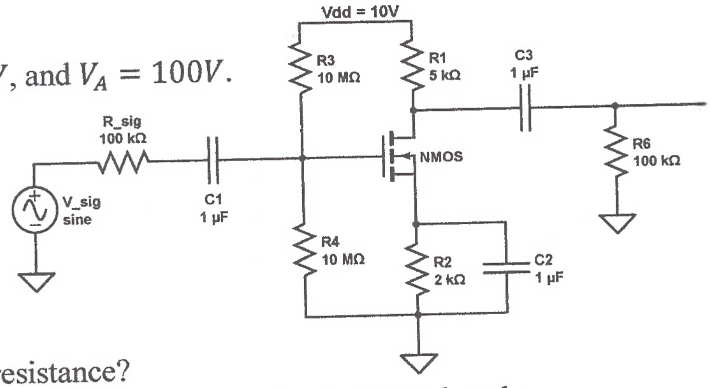
Upward swing : $v(+) = V_{DD} - V_D = 2.5V$

Downward " $v(-) = V_D - V_{D,sat} = 2.5 - V_{OV} = 1.5V$

Overall swing $= \pm 1.5V$

$\Rightarrow$ BJT Amplifier has larger output swing

3. The NMOS below has $k_n = 1\ mA/V^2$, $V_{tn} = 1V$, and $V_A = 100V$.
   a. What is the amplifier configuration?
   b. Find the DC bias current and drain-source voltage. You can assume $V_A = \infty$ for DC analysis.
   c. Find the small signal parameters, g_m, r_o.
   d. What is the overall gain of the amplifier including the signal resistance and load resistance?
   e. What is the maximum output voltage swing? (*Note: output swing is defined as the smaller of the upward and downward voltage swings*)



(a) Common Source Amplifier

(b) $I_D = \frac{1}{2} k_n (V_{GS} - V_{tn})^2 = \frac{V_S}{R_2}$

$V_{GS} = V_G - V_S$

$V_G = V_{dd} \cdot \frac{R_4}{R_3 + R_4} = 5V$

$\Rightarrow \frac{1}{2} \cdot 1 \cdot (4 - V_S)^2 = \frac{V_S}{2} \Rightarrow V_S^2 - 8V_S + 16 = V_S$

$\Rightarrow V_S = 2.44\ V$ (the other solution of 6.6 V leads to cut-off of MOS)

$I_D = 1.22\ mA$ # $\cdot V_D = 10 - I_D \cdot 5k\Omega = 3.9V$. $V_{DS} = 1.46V$ #

↑ See note at the end of the solution

(c) $g_m = \frac{2 I_D}{V_{ov}} = \frac{2.44}{(5 - 2.44 - 1)} = 1.56\ mS$ #

$r_o = \left(\frac{I_D}{V_A}\right)^{-1} = 82\ k\Omega$ #

(d) $G_v = \frac{-R_{in}}{R_{sig} + R_{in}} \cdot A_{vo} \cdot \frac{R_L}{R_o + R_L} = 0.98 (-g_m)(R_1 // R_6 // r_o)$, where $R_1 // R_6 // r_o = 4.5k\Omega$

$R_{in} = R_3 // R_4 = 5\ M\Omega$

$G_v = -6.87\ V/V$

(Continuation of Problem 3 solution)

(e) $V_D = 3.9\,V$

Upward swing = $V_{DD} - V_D = 10 - 3.9 = 6.1\,V$

Downward swing = $3.9 - 4 = -0.1$

↑ $V_G - V_{tn} = 4\,V$

⇒ No swing

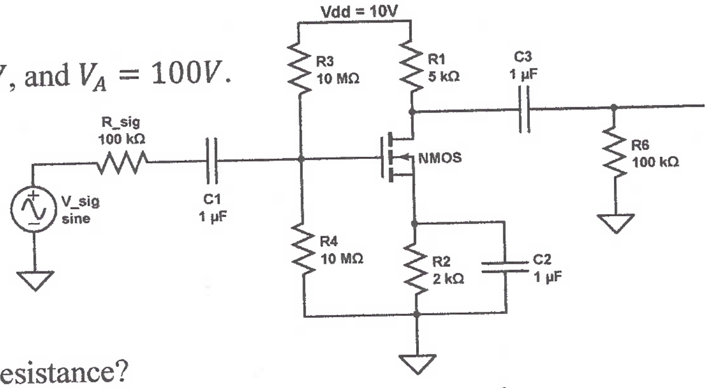
(The circuit is not well designed!)

The MOSFET is not in Saturation.

$V_{DS} < V_{ov}$

** Corrected Problem **

3. The NMOS below has $k_n =$ ~~$1\,mA/V^2$~~ $0.5\,mA/V^2$, $V_{tn} = 1V$, and $V_A = 100V$.
   a. What is the amplifier configuration?
   b. Find the DC bias current and drain-source voltage. You can assume $V_A = \infty$ for DC analysis.
   c. Find the small signal parameters, g_m, r_o.
   d. What is the overall gain of the amplifier including the signal resistance and load resistance?
   e. What is the maximum output voltage swing? (*Note: output swing is defined as the smaller of the upward and downward voltage swings*)



(a) Common Source

(b) $I_D = \frac{1}{2} k_n (V_{GS} - V_{tn})^2 = \frac{V_S}{R_2}$, $V_{GS} = V_G - V_S = V_{DD}\frac{R_4}{R_3+R_4} - V_S$

$\Rightarrow \frac{1}{2}\cdot\frac{1}{2}\cdot(5 - V_S - 1)^2 = \frac{V_S}{2}$

$\Rightarrow (4 - V_S)^2 = 2V_S \Rightarrow V_S^2 - 8V_S + 16 = 2V_S$

$V_S = 2V$ (8V is unphysical)

$I_D = 1\,mA$

$V_{DS} = V_{DD} - I_D(R_1 + R_2) = 3V > V_{OV} = V_{GS} - V_{tn} = (5-2) - 1 = 2V$

(c) $g_m = \frac{2I_D}{V_{OV}} = \frac{2}{2} = 1\,mS$

$r_o = \frac{V_A}{I_D} = 100\,k\Omega$

(d) $G_v = -\frac{R_{in}}{R_{sig} + R_{in}}(g_m)(R_1 /\!/ r_o /\!/ R_6) = -4.4\ V/V$

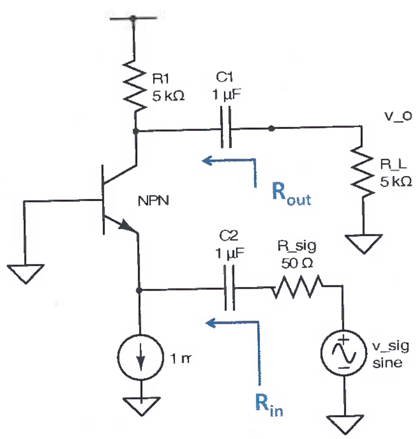
$R_{in} = R_3 /\!/ R_4 = 5\,M\Omega$

(e) $v(+) = V_{DD} - V_D = I_D R_D = 5V$

$v(-) = V_D - V_{OV} = 5 - 2 = 3V$

$\Rightarrow$ Output swing $= \pm 3V$.

4. The NPN BJT has a current gain of 100. Assume Early voltage $V_A = \infty$.
   a. What is the amplifier configuration?
   b. What is the input resistance (not including R_L)?
   c. What is the output resistance, R_{out} (including R_{sig})?
   d. What is the overall voltage gain, including the signal and load resistance?
   e. What is the overall current gain of the amplifier, i.e., the ratio of the current flowing R_L and current from source?



(a) Common base

(b) $R_{in} = r_e \approx g_m^{-1} = \boxed{26\,\Omega}$

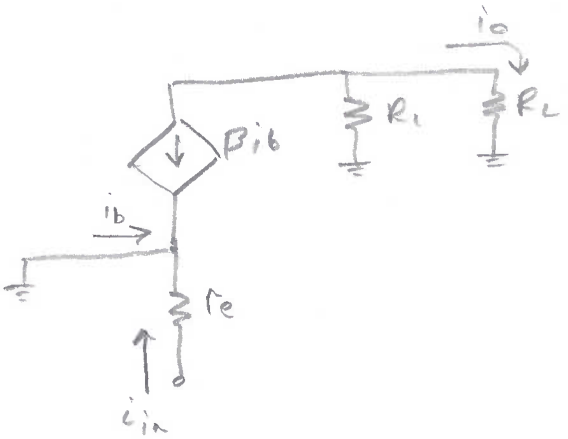
(c) $\boxed{R_{out} = 5\,k\Omega}$

(d) $G_v = \dfrac{R_{in}}{R_{sig} + R_{in}} \, g_m (R_1 // R_L)$

$= \dfrac{26}{50 + 26} (0.04)(2.5k)$

$\boxed{G_v = 34 \text{ V/V}}$

(e) $G_i = \dfrac{R_{sig}}{R_{in} + R_{sig}} A_i \qquad A_i = \dfrac{i_o}{i_{in}}$



$i_o = -\beta i_b \left(\dfrac{R_1}{R_L + R_1} \right)$

$i_{in} = -(\beta + 1) i_b$

$i_o = i_{in} \left(\dfrac{\beta + 1}{\beta} \right) \left(\dfrac{R_1}{R_L + R_1} \right)$

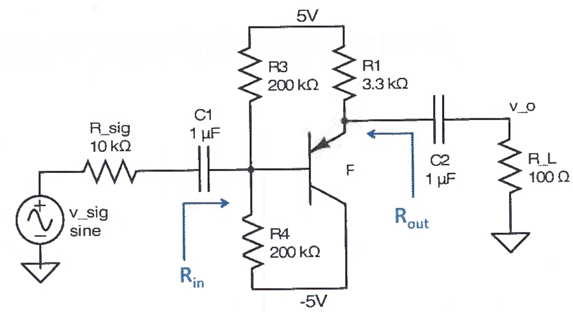
$A_i = \dfrac{i_o}{i_{in}} \approx \dfrac{R_1}{R_L + R_1} = \dfrac{5k}{5k + 5k}$

$\boxed{A_i = 0.5 \text{ A/A}}$ Full credit was given for this answer

The question asked for overall current gain need to consider source resistance.

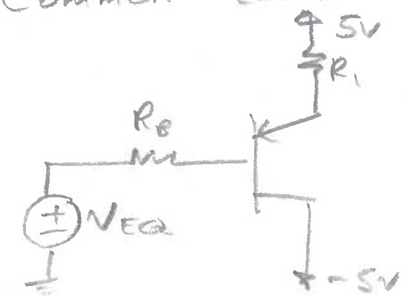
$G_i = \dfrac{R_{sig}}{R_{in} + R_{sig}} A_i = \dfrac{50}{50 + 26} (0.5) = 0.33 \text{ A/A}$

5. The PNP BJT has a current gain of 100. Ignore Early effect.
   a. What is the amplifier configuration?
   b. Find the DC bias point (collector current and collector-to-emitter voltage).
   c. What is the input resistance, R_{in} (not including R_L)?
   d. What is the output resistance, R_{out} (including the effect of R_{sig})?
   e. What is the overall voltage gain including signal and load resistances?



(a) Common collector (emitter follower)

(b)



$V_{EQ} = 0$

$R_B = 200k // 200k$

$= 100k\Omega$

$5V - I_E R_1 - V_{EB} - I_B R_B = 0$

$5V - I_E R_1 - V_{EB} - \frac{I_C}{\beta} R_B = 0$

$\approx 5V - I_C R_1 - V_{EB} - \frac{I_C}{\beta} R_B = 0$

$I_C \left[-R_1 - \frac{R_B}{\beta} \right] = -4.3$

$I_C = 1\,mA$

$V_{CE} = -(5 - 3.3 + 5V)$

$V_{CE} = -6.7\,V$

(c) $R_{in} = (\beta+1)(r_e + R_1) // R_B$

$= (101)(26 + 3300) // 100k$

$R_{in} = 77\,k\Omega$

(d) R_{in} (including R_L) $= (\beta+1)(r_e + R_1//R_L) // R_B$

$= (101)(26 + 97) // 100k$

$= 11\,k\Omega$

$G_v = \frac{R_{in}}{R_{sig} + R_{in}} \cdot \frac{R_L // R_1}{R_L // R_1 + r_e} = \left(\frac{11k}{11k + 10k} \right) \left(\frac{97}{97 + 25} \right)$

$= 0.42\ V/V$