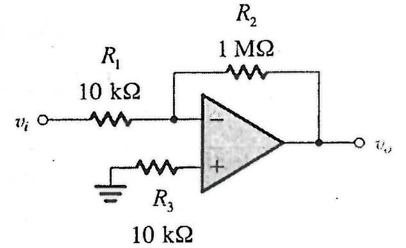
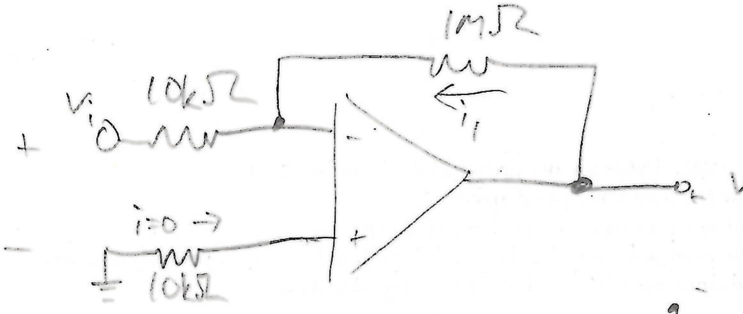
1. Consider the Op Amp circuit on the right.
   8 a. Derive the voltage gain expression of the amplifier if we consider the Op Amp to be ideal (infinite open-loop gain, infinite input resistance, zero output resistance)?
   6 b. What is the input resistance of the amplifier?
   6 c. What is the value of the voltage gain for a signal with 10 kΩ source resistance?



a.)



$$V_+ = 0 \Rightarrow V_- = 0$$

$$i_1 = \frac{V_o - 0}{1M\Omega} = \frac{-(V_i - 0)}{10k\Omega}$$

$$\Rightarrow \frac{V_o}{V_i} = -\frac{1M\Omega}{10k\Omega} = -100\ (V/V)$$

$$\boxed{\frac{V_o}{V_i} = -100\ (V/V)}$$ ⑧

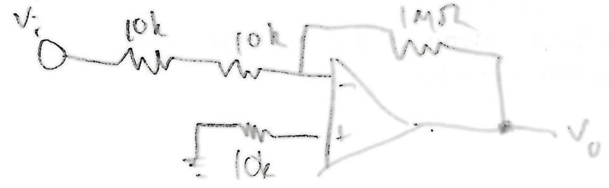
final numbers account for 2 points each sub question

b.)

$$i_i = \frac{V_i}{10k} \Rightarrow R_{in} \equiv \frac{V_i}{i_i} = 10k\Omega$$

$$\boxed{R_{in} = 10k\Omega}$$ ⑥

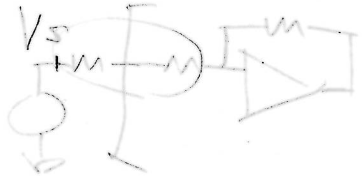
c.)



$$\frac{V_o}{V_i} = A_v\left(\frac{R_{in}}{R_{in}+R_s}\right) = -100 \cdot .5 = -50$$

$$\Rightarrow \boxed{\frac{V_o}{V_i} = -50\ (V/V)}$$ ⑥

2. For the same circuit as in Problem 1 and the same source resistance, but consider an Op Amp with a finite open loop gain of $A_0 = 100{,}000$, and an open-loop bandwidth of 100 Hz. The input resistance of the Op Amp is still infinite, and the output resistance is zero.
   10 a. What is the 3-dB frequency of the amplifier in unit of Hz?
   (You don't need to re-derive the expression if you can deduce the frequency response of the close-loop amplifier).
   10 b. Show the frequency response in Bode plot. Mark the 3-dB frequency, unity gain frequency, the low frequency gain in dB, and the slope in dB/decade.



a.) Fixed gain-bandwidth product

$|A| \cdot f_{3dB} = 10^5 \cdot 10^2 \text{ Hz} = 10^7 \text{ Hz}$

Amplifier in Prob. 1 $|A| = 50$

-5 if neglect source resistance

$\Rightarrow f_{3dB} = \frac{10^7}{|A|} = \frac{10^7}{50} = 2 \cdot 10^5 = 200 \text{ kHz}$

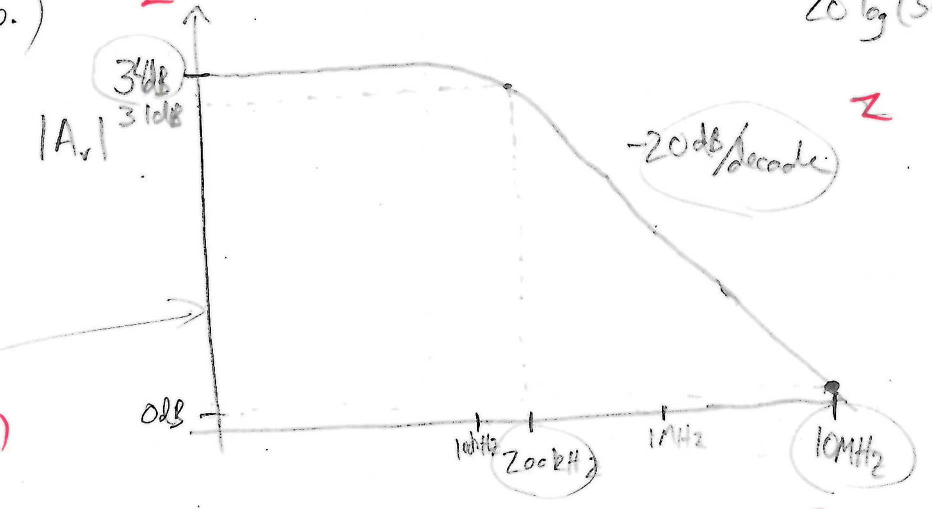
$\boxed{f_{3dB} = 200 \text{ kHz}}$

$\frac{10^7}{100} = 10^5 \text{ Hz}$
100 kHz

5

b.) Z

$20 \log(50) \approx 34 \text{ dB}$

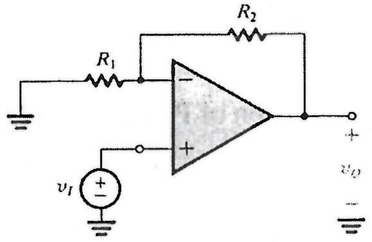


Z

Z (shape)

Z Z

Half points deducted for carryover from part a)



3. A non-inverting amplifier with variable gain is shown on the right. Here, $R_1 = 1$ kΩ and R_2 is variable from 100 kΩ to 100 MΩ. This input signal is:
   $v_i(t) = (10mV) \cdot \sin(2\pi \cdot 10kHz \cdot t)$.
   The Op Amp has a slew rate (SR) of 6.28 V/μs.
   a. First, assume the Op Amp has infinite bandwidth. If we gradually increase the gain of the amplifier by increasing R₂, at what value of R_2 does the output become slew rate limited?
   b. Now consider the Op Amp with a finite gain-bandwidth product of 100 MHz and R_2 = 20 MΩ, is the amplifier bandwidth-limited or SR-limited? Show the calculation supporting your answer.

a. Gain $A_v = 1 + \frac{R_2}{R_1}$ account for 3pt

$V_o = A_v \cdot v_i$

$$\left.\frac{dV_o}{dt}\right|_{max} = A_v \cdot 10mV \cdot 2\pi \cdot 10k \cdot \cos(2\pi \cdot 10k\,t)\Big|_{2\pi \cdot 10k \cdot t = 0}$$

accounts for 4pt

$$= A_v \cdot 10mV \cdot \cancel{2\pi \cdot 10k} \le SR = 6.28V/\mu s$$
$$= \cancel{2\pi \cdot 1000k}\ 10^2$$

final number 2

$$A_v \le \frac{10^2}{0.01} = 10^4 \Rightarrow R_2 \lesssim 10M\Omega$$ ⑩

b. Given $R_2 = 20M\Omega$. $A_{v_o} = 1 + \frac{20M}{1k} \cong 2\times10^4$ (with 10^3 written over the fraction)

account for 3 points

$$f_{3dB} = \frac{100M}{2\times10^4} = \boxed{5kHz} < 10kHz$$ (with 10^2 written over $100M$)

At first, the Op Amp is $\boxed{\text{BW limited}}$ ⑤

then, consider the BW caused attenuation

if answer SR limited (with math) but forget BW attenuation penalty -2

$$A_v\Big|_{@10kHz} = \left.\frac{A_{v_o}}{\sqrt{1+\left(\frac{f}{f_{3dB}}\right)^2}}\right|_{f=10kHz} = \frac{2\times10^4}{\sqrt5}$$

$$\text{So}\ \left.\frac{dV_o}{dt}\right|_{max} = \frac{2\times10^4}{\sqrt5}\times 10mV \cdot 2\pi \cdot 10k < SR$$ (with "<1" under $\frac{2\times10^4}{\sqrt5}$)

$\boxed{\text{NO SR limited}}$ ⑤

4. A pn junction in the reverse-bias configuration can be used as a variable capacitor. In the lab, we measured the capacitances of a pn junction diode at two voltages:
   (1) At 2.4 V reverse bias, the capacitance is measured to be 0.5 pF
   (2) At 12 V reverse bias, the capacitance is measured to be 0.25 pF
   10 a. Find the capacitance of the pn junction diode at zero bias.
   10 b. Find the built-in potential of the pn junction diode.

Given: $C_{jo} = A\sqrt{\left(\frac{\epsilon_s q}{2}\right)\frac{N_A N_D}{(N_A+N_D)}} \cdot \frac{1}{\sqrt{V_0}}$

$$= \frac{\alpha}{2\sqrt{V_0}}$$

We know with reverse bias $V_0 \Rightarrow V_0 + V_R$

$$\Rightarrow C_j = \frac{\alpha}{2\sqrt{V_0+V_R}}$$

$$\Rightarrow C_j = \frac{C_{jo} \cdot \sqrt{V_0}}{\sqrt{V_0+V_R}} = \frac{C_{jo}}{\sqrt{1+\frac{V_R}{V_0}}}$$

$$C_j^2\left(1+\frac{V_R}{V_0}\right) = C_{jo}^2$$

@ $V_R = 2.4\,V$, $C_j = 0.5\,pF$

$$\Rightarrow .25\left(1+\frac{2.4}{V_0}\right) = C_{jo}^2$$

@ $V_R = 12V$, $C_j = 0.25\,pF$

$$\Rightarrow .0625\left(1+\frac{12}{V_0}\right) = C_{jo}^2 = .25\left(1+\frac{2.4}{V_0}\right)$$

−2 if close w/ correct approach
−5 if correct approach but not close

$$1+\frac{12}{V_0} = 4+\frac{9.6}{V_0} \Rightarrow V_0 + 12 = 4V_0 + 9.6$$

$$\Rightarrow 3V_0 = 2.4, \boxed{V_0 = 0.8\,V}$$

10

$$C_{jo}^2 = .25\left(1+\frac{2.4}{.8}\right) = 1. \Rightarrow \boxed{C_{jo} = 1pF}$$

10

5. Consider the following two one-sided pn junction diodes with opposite doping profiles:
Diode A: $N_D = 10^{18}$ cm^{-3}, $N_A = 10^{16}$ cm^{-3}
Diode B: $N_D = 10^{16}$ cm^{-3}, $N_A = 10^{18}$ cm^{-3}
The rest of the material parameters are shown in the table. Other parameters (e.g., junction area) of the two diodes are identical.

Electron diffusion coefficient	D_n	30 cm^2/s
Hole diffusion coefficient	D_p	10 cm^2/s
Electron diffusion length	L_n	2 μm
Hole diffusion length	L_p	1 μm

8 a. What is the *ratio* of the current going through the diodes under a forward bias voltage of 0.8V?

6 b. What is the *difference* of the built-in voltages between these two diodes?

6 c. What is the *ratio* of the diode capacitances at zero bias?

8

a.) Diode current eqn: $I = Aq n_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)\left(e^{V/V_T} - 1\right)$

Diode A: $I_A \simeq I_n = Aq n_i^2 \frac{D_n}{L_n N_{A_A}}\left(e^{V/V_T} - 1\right)$

Diode B: $I_B \simeq I_p = A_q n_i^2 \frac{D_p}{L_p N_{D_B}}\left(e^{V/V_T} - 1\right)$

$\Rightarrow \frac{I_A}{I_B} = \frac{D_n}{D_p} \cdot \frac{L_p}{L_n} \cdot \frac{N_{D_B}}{N_{A_A}} = \frac{30}{10} \cdot \frac{1}{2} \cdot \frac{\cancel{10^{16}}}{\cancel{10^{16}}} = \boxed{\frac{3}{2}}$

~~EACH~~ wrong number take 2 points off

6

b.) $V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$ $N_A N_D = 10^{16} \cdot 10^{18} = 10^{34}$ for both diodes

$\Rightarrow V_{0A} = V_{0B} \Rightarrow V_{0A} - V_{0B} = \boxed{0V}$

6

c.) $C_{j0} = A\sqrt{\frac{\epsilon_s q}{2} \frac{N_A N_D}{N_A + N_D}} \cdot \frac{1}{\sqrt{V_0}}$ $\frac{N_A N_D}{N_A + N_D}$ is the same for both A and B

$V_{0A} = V_{0B}$

$\Rightarrow C_{j0A} = C_{j0B}$

$\Rightarrow \frac{C_{j0A}}{C_{j0B}} = \boxed{1}$