MIDTERM 1 Fall-2015

Instructor: Prof. A. LANZARA

TOTAL POINTS: 100

Show all work, and take particular care to explain what you are doing. Partial credit is given. Please use the symbols described in the problems, define any new symbol that you introduce and label any drawings that you make. If you get stuck, skip to the next problem and return to the difficult section later in the exam period.
All answers should be in terms of variables.

GOOD LUCK!

PROBLEM 1 (total 10pts)

A glass marble R in radius is to be dropped through a hole in a steel plate. At temperature T_0 the hole radius is smaller than the marble radius and is equal to 2/3 R. By how much must the steel temperature be raised so that the marble will fit through the hole? The coefficient of linear expansion of steel is α.
Hint: remember the demo we did in class with the steel ring and ball.

PROBLEM 2 (total 10pts)

A lead bullet of mass m, specific heat c_B, and temperature T_B is fired at a speed v_B into a large block of ice at T=0°C in which it becomes embedded. The heat of fusion of ice is L. What quantity of ice melts?

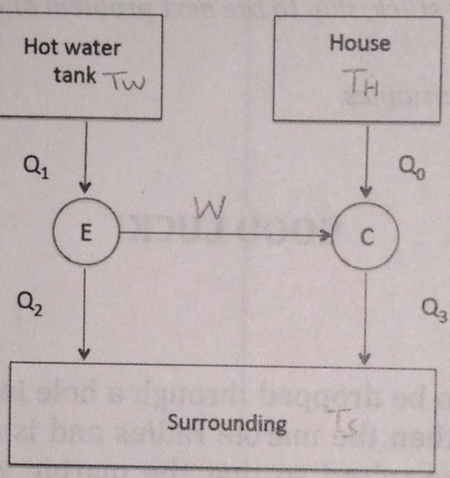
PROBLEM 3 (total 20pts)
A one mole of an ideal diatomic gas, originally at pressure P_0, undergoes a four step process: first it expands at constant pressure from A to B; then it expands isothermally from B to C; then it contracts isobarically from C to D; and finally it is compressed adiabatically from D to A. Assume that at point A the gas is at pressure P_A and volume V_A, at point B the temperature is T_B, and at point C the pressure is P_C.
a) (3pts) Plot these processes on a PV diagram
b) (5pts) Determine T_C and T_D in terms of the variables given.
c) (12pts) Calculate the change in internal energy, the work done by the gas, and the heat added to the gas for the cycle.

PROBLEM 4 (total 20pts)

Consider the air conditioning of a house through use of solar energy. At a particular location, the flux of solar radiation allows a large tank of water to be maintained at a temperature T_W. During a particular time interval, heat in the amount of Q_0 must be extracted from the house to maintain its temperature at T_H when the surrounding temperature is T_S.

The process is schematically represented below.

Treating the tank of water, the house, and the surroundings as heat reservoirs, determine the minimum amount of heat that must be extracted from the tank of water by any device to accomplish the required cooling of the house. No other sources of energy are available.



PROBLEM 5 (total 20pts)

Consider a mole of a gas initially at $1 \equiv (P_1, V_1, T)$ and finally at $2 \equiv (P_2, V_2, T)$.

a) (5pts) Choose the simplest one-step thermodynamic path to go from 1 to 2 and draw it in the PV diagram. Calculate the entropy change of the gas $\Delta S = S_2 - S_1$.

Let's now consider a different path to go from 1 to 2. In this new path, you first change pressure at constant volume and then volume at constant pressure, passing through an intermediate point 0 $\equiv (P_0, T_0)$.

b) (5pts) Draw this new path in the PV diagram. What is the entropy change in each step of the path? Explain and show your answer.

c) (5pts) Let's assume now that 1 and 3 lie on an adiabatic curve with $1 \equiv (P_1, V_1)$ and $3 \equiv (P_3, V_3)$. What would the change in entropy be in this case? Explain your answer.

d) (5pts) If the gas is undergoing a free expansion from 1 to 2, what is the entropy change in this case? Explain your answer.

PROBLEM 6 (total 10pts)

Two moles of an ideal gas with molar specific heat $c_v = 5/2\ R$ are initially at temperature
pressure P_0. Vibrational degrees of freedom can be neglected in this temperature range.

a) (3pts) How many degrees of freedom does the gas have? Explain your answer.
b) (2pts) Could this be a monatomic gas? Explain your answer.
c) (5pts) Determine the change in internal energy, the temperature change and work
the gas when heat Q is added to the gas at constant pressure.

$$C_P - C_V = R = N_A k$$

$$\gamma = \frac{C_P}{C_V} = \frac{d+2}{d}$$

$$C_V = \frac{d}{2} R$$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n! 2^{2n+1}} \sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int (1+x^2)^{-1/2} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2} \ln(1+x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha-1)\alpha}{2} x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$