University of California, Berkeley
Physics H7A Fall 2014

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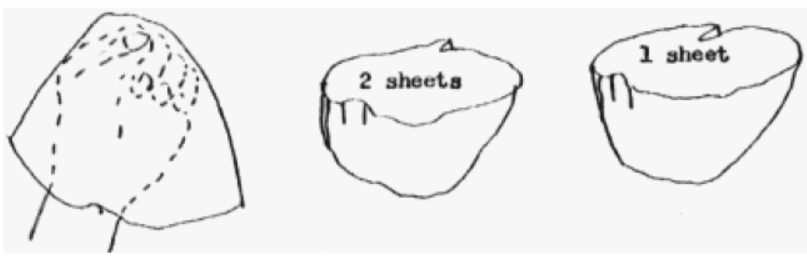
Solutions to MIDTERM EXAM 1

Tuesday October 7, 9.40 - 11.00 am

Maximum score: 150 points

1. *(xx points) To determine the dependence of the force of air resistance upon the speed of a slowly moving object, one starts with three sheets of paper stacked together as they come out of the package. One then crumples them against ones fist as shown in the sketch. Next one separates a single sheet from the other two without changing their*



crumpled (pseudo-conical) shapes. This yields two objects with the same shape, but with different masses: m and $2m$, where m is the mass of a single sheet.

Finally one releases these two objects simultaneously, and compares their motion in still air under the influence of gravity.

(a) *Instantaneously after the two objects are released, what is the ratio R of the downward acceleration of the heavier object to that of the lighter object?*

(b) *Very soon after being released, the two objects reach terminal (constant) velocity due to the effects of air resistance. After a long time, one observes that the object of mass $2m$ has dropped $\sqrt{2}$ times farther than the object of mass m. Assuming that the force of air resistance on these objects is proportional to v^α, where v is the velocity and α is a constant exponent, what is the value of α?*

(c) *Suppose that Mother Nature were to turn gravity off while these objects are falling. If one were willing to wait an arbitrarily long time, would they fall an arbitrarily long distance, or would that distance be bounded? Explain. (Note: use the value of α you found in part (b)).*

1. Solution

What do we know?

- Two objects
- Same shape

- Different masses (m, $2m$)
- Forces present:
  - Gravity: $F_g = mg$
  - Air resistance: $F_a \propto v^\alpha$

Part (a)
The questions asks for the downward acceleration "immediately after the objects are released".

- We know they start from rest.
- Immediately after release the velocity must still be negligible, or they would have experienced ∞ acceleration.
- Air resistance is proportional to velocity, therefore F_a must also be negligible
- Therefore the only force is gravity
- Therefore they each experience acceleration g
- Therefore the ratio of accelerations is $R = 1$

Part (b)

- The objects are now traveling at terminal velocity.
- They are therefore no longer accelerating.
- Therefore, the two forces must balance.
- Let's take the constant of proportionality in F_a to be K, then we have two equations:

$$2mg = Kv_2^\alpha \quad (1)$$
$$mg = Kv_1^\alpha \quad (2)$$

- Take the ratio of the two $\rightarrow$:

$$2 = \frac{v_2^\alpha}{v_1^\alpha} \quad (3)$$
$$= \left(\frac{v_2}{v_1}\right)^\alpha \quad (4)$$
$$2^{1/\alpha} v_1 = v_2 \quad (5)$$

- We are told they reach terminal velocity "very soon" after being released.
- We are are then told the ratio of distances "after a long time".
- We can therefore assume that the period of acceleration at the start was negligible, and the distance travelled is proportional to the terminal velocity

- This tells us:

$$v_2 = \sqrt{2} v_1$$

- Combined with the previous equation, this tells us

$$\alpha = 2$$

Part (c)

- If we turn off gravity, then the only force remaining is air resistance
- We therefore have:

$$m\frac{dv}{dt} = -Kv^2$$

- We can separate variables by dividing through by v^2, multiplying by dt, and integrating:

$$\int -\frac{1}{v^2} = \int \frac{K}{m} dt \quad (6)$$
$$\frac{1}{v} = \frac{K}{m} t + C \quad (7)$$

  Where C is a constant of integration.

- "An arbitrarily long time" means we can take $t \to \infty$, therefore C is negligible next to $\frac{K}{m} t$
- Therefore as $t \to \infty$, $v \propto 1/t$
- We can integrate this statement to find the distance fallen, and see that $x \propto \ln t$
- $\ln t$ is arbitrarily large for arbitrarily large t, therefore the distance is unbounded

2. *(xx points) The Voyager 1 spacecraft, launched on August 20, 1977, has left the Solar system, and is now traveling in the interstellar medium of the galactic disk. There is a nonzero density of matter in the disk, however, and the spacecraft will start slowing down. Take the average density of matter in the disk to be roughly* 3×10^{-21} *kg/m*^{3}*, and calculate the distance that the spacecraft will have to travel for the velocity to decrease to half the velocity with which it left the Solar System. The mass of the spacecraft is 722 kg, and approximate the cross-sectional area of the spacecraft to be 1 m*^{2}*. Assume that the matter in the galactic disk is made up of cold dust, meaning that the particles in the disk have zero velocity. Assume also that the dust collides elastically with the spacecraft so that in the spacecrafts frame*

$$v^{dust}_{\text{before collision}} = -v^{dust}_{\text{after collision}}$$

Neglect gravitational forces on the spacecraft. You may wish to use

$$\frac{dv}{dt} = v\frac{dv}{dx}$$

2. Solution

What do we know?

- Spacecraft of mass m (plug numbers in at the end!), cross-sectional area A
- Dust of density ρ, at speed $v_d^{init} = 0$ (in frame of an outside observer)
- Therefore, in the spacecraft's frame:

$$v_d^i = -v_s$$

$$v_d^f = v_s$$

  Therefore

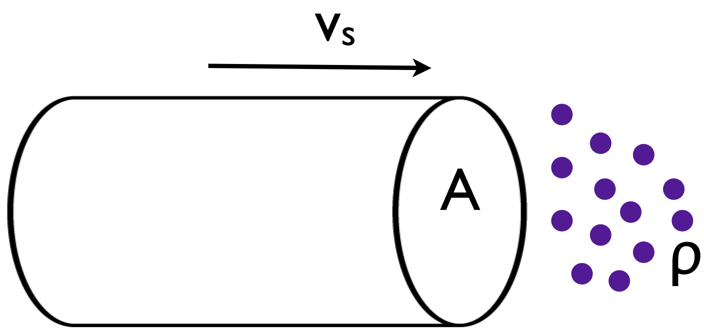
$$\Delta v_d = 2v_s$$

- The momentum imparted to the dust is therefore

$$\Delta p_d = m_d 2v_s$$

- Of course, this depends on:
  - The instantaneous velocity of the ship;
  - The rate at which the ship collides with dust (thus how much m_d we are considering).
- The amount of dust colliding with the ship is given by:

$$\frac{dm_d}{dt} = \rho A v_s$$



- Therefore the rate of change of the momentum is

$$\frac{dp}{dt} = \frac{dm_d}{dt} 2v_s = 2\rho A v_s^2$$

- The change in momentum of the dust is equal to the force exerted on the dust
- The force exerted on the ship is equal and opposite (N3)
- The force on the ship will cause it to slow (N2):

$$F_s = -F_d = -2\rho A v_s^2 = m_s \frac{dv_s}{dt}$$

- We are given an equation, so let's assume it will help us and use it now:

$$\frac{dv}{dt} = v\frac{dv}{dx}$$

Which gives us

$$-2\rho A v_s^2 = m_s \frac{dv_s}{dt} = m_s v_s \frac{dv_s}{dx}$$

- Maths:

$$-2\rho A v_s^2 = m_s v_s \frac{dv_s}{dx} \tag{8}$$

$$\int_0^{x_f} -2\rho A dx = m_s \int_{v_i}^{v_f} \frac{1}{v_s} dv_s \tag{9}$$

$$-2\rho A x_f = m_s \ln \frac{v_f}{v_i} \tag{10}$$

- We are looking for x when $v_f = 0.5 v_i$

$$-2\rho A x_f = m_s \ln \frac{v_f}{v_i} \tag{11}$$

$$x_f = -\frac{m_s}{2\rho A} \ln \frac{1}{2} \tag{12}$$

$$= \frac{m_s}{2\rho A} \ln 2 \tag{13}$$

- NOW plug in numbers! $x_f = 8.34 \times 10^{22}$m

3. *(points) A wooden block of mass M, initially at rest on a horizontal table with coefficient of sliding friction μ, is struck by a bullet of mass m and velocity v. The bullet lodges in the center of the block. How far does the block slide?*

3. Solution

Instantaneously after the collision of the bullet and block, after the bullet has come to rest but before the frictional force on the block has had time to slow it down more than an infinitesimal amount, we can apply momentum conservation to the bullet-block collision. At that time the total momentum of the block+bullet system is $(M+m)v'_0$, where v'_0 is the velocity of the block+bullet system immediately after the collision. Momentum conservation requires that momentum to be equal to the initial momentum mv of the bullet. Thus

$$v'_0 = \frac{mv}{M+m} .$$

After the collision, the normal force on the block+bullet system from the table is $(M+m)g$, giving rise to a frictional force

$$\mu N = \mu(M+m)g$$

on the sliding block+bullet system. This causes a constant acceleration μg of that system opposite to its velocity.

Take $t = 0$ at the time of collision. Afterward, the block+bullet system's velocity in the horizontal direction will be $v'(t) = v'_0 - \mu g t$. It will continue sliding until $v'(t) = 0$, at which point the frictional force will disappear and it will remain at rest. Solving, the time at which the block-bullet system stops is

$$t = v'_0/(\mu g) .$$

The distance traveled in that time is

$$x = v'_0 t - \frac{1}{2}\mu g t^2 = \frac{1}{2} v'_0 t = \frac{(v'_0)^2}{2\mu g} .$$

Plugging in the already deduced value for v'_0, this distance is

$$x = \Big(\frac{m}{M+m}\Big)^2 \frac{v^2}{2\mu g} .$$

4. *(20 points) An asymmetric barbell stands vertically at rest on frictionless horizontal ice. Mass M rests on the ice, and mass m is a distance h above it; the mass of the bar that rigidly connects these two masses is negligible. The dimensions of the masses can be neglected in comparison to their separation h. Take $x = 0$ to be the initial position of mass M. Mass m is given an infinitesimal tap in the $+\hat{\mathbf{x}}$ direction that produces a negligible momentum, but does eventually cause the barbell to topple over. You may assume that mass M remains in contact with the ice throughout the motion. When mass m hits the ice, at what horizontal coordinate x_M will mass M be located?*

4. Solution

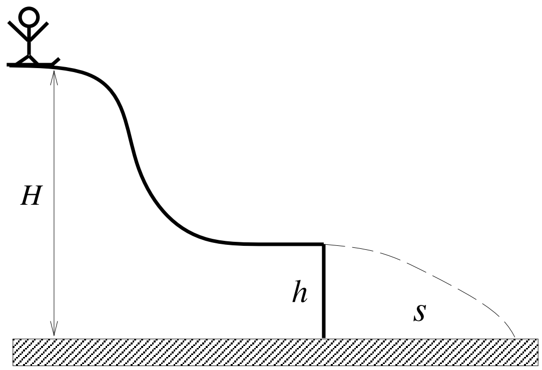
Since the ice is horizontal and frictionless, it cannot exert any force on the barbell in the $\hat{\mathbf{x}}$ direction, either as the result of a contact force or a frictional force. Therefore the x coordinate x_{CM} of the barbells center of mass, initially at rest at $x = 0$, must remain at $x = 0$. When mass m hits the ice and the barbell is horizontal,

$$x_{CM} = 0 \tag{14}$$

$$= \frac{Mx_M + m(x_M + h)}{M + m} \tag{15}$$

$$\rightarrow x_M = -h\frac{m}{M + m} \tag{16}$$

5. *(30 points)* Ski jumping *is a nordic sport to be featured in the upcoming Winter Olympics. Imagine an athlete sliding down a ski jump of height H, starting at rest. The jump is horizontal near the release point (see picture). Ignoring friction, what height of the jump h would provide the maximum jump distance $s_{\max}$, and what is that distance ?*



5. Solution

This problem combines the concepts of energy conservation with kinematics, which we studied at the beginning of the semester.

First, let's express distance s in terms of parameters H and h, and then find when s is maximum. Suppose velocity of the skier at the release point is $\mathbf{v}$. The velocity is horizontal. After the release, the skier will move under the influence of gravity, and his x and y coordinates will depend on time as

$$x(t) = vt \tag{17}$$

$$y(t) = h - \frac{gt^2}{2}. \tag{18}$$

Plugging in $x(t) = s$ and $y(t) = 0$ (condition for landing), we get

$$s = v\sqrt{\frac{2h}{g}}.$$

Now we need to find the velocity v at the release point. We get it from conservation of energy:

$$\frac{mv^2}{2} = mgH - mgh$$

where m is the skier mass. Therefore,

$$s = 2\sqrt{h(H-h)}.$$

In order to find the maximum distance, we need to require that the first derivative of s *w.r.t.* to h (the only variable in the problem) is zero:

$$\frac{ds}{dh} = \frac{H-2h}{\sqrt{h(H-h)}} = 0$$

The solution is

$$h = H/2 \tag{19}$$

$$s_{\max} = H. \tag{20}$$