Problem #1: Huang MT1 F08

Given: $l_m = 5.6 \times 10^{-8}$ m at STP

Find: d_{CO_2}

Solution: Use $l_m = \dfrac{1}{4\pi\sqrt{2}\, r^2 (N/V)}$

[One common mistake: using the formula $l_m = \dfrac{1}{4\pi r^2 (N/V)}$

Note: V = volume, not velocity]

Solve for r:

$$\left(l_m = \frac{1}{4\pi\sqrt{2}\, r^2 (N/V)}\right)\frac{r^2}{l_m}$$

$$r^2 = \frac{1}{4\pi\sqrt{2}\, l_m (N/V)}$$

$$r = \sqrt{\frac{1}{4\pi\sqrt{2}\, l_m (N/V)}}$$

$$\Rightarrow \quad d = 2r = 2\sqrt{\frac{1}{4\pi\sqrt{2}\, l_m (N/V)}}$$

$$= \sqrt{\frac{1}{\pi\sqrt{2}\, l_m (N/V)}}$$

Give diameter, not radius as answer

Now, need to find N/V:

Method 1: 1 mol occupies 22.4L at STP and 1 mol = 6.02×10^{23} mol.

However, V has to be in units of m^3 not L, $V = 0.0224\ m^3$

$$\frac{N}{V} = \frac{6.02 \times 10^{23}\ mol}{0.0224\ m^3} = 2.68 \times 10^{25}\ mol/m^3$$

Method 2: $PV = NkT \Rightarrow \frac{N}{V} = \frac{P}{kT}$

At STP, P = 1 atm but we need to convert this to N/m^2: $P \approx 10^5\ N/m^2 (Pa)$

$$\frac{N}{V} = \frac{10^5\ N/m^2}{1.38 \cdot 10^{-23}\ J/K \cdot 273\ K} = 2.65 \times 10^{25}\ mol/m^3$$

Note: Both T = 273 K & T = 293 K were accepted

$$d = \sqrt{\frac{1}{\pi\sqrt{2} \cdot 5.6 \cdot 10^{-8} \cdot 2.68 \times 10^{25}}} = 3.87 \cdot 10^{-10}\ m$$

(1 atm, 273K)

Also acceptable

$d = 4.0 \cdot 10^{-10}\ m$ (1 atm, 293K)

Problem # 2: Huang MT1 F08

i) D: $P = \varepsilon \sigma A T^4$

ii) A: From class

iii) D: $P(\text{all heads}) = \frac{1}{2^{1000}}$ so need to throw it many times to achieve this microstate

iv) C: Carnot cycles & isothermal expansions are always reversible
a) is reversible because any chemical equation has some non-zero probability of reversing.

v) C: $l_f = l_0(1 + \alpha \Delta T)$

$A = l_f^2 = l_0^2 (1 + \alpha \Delta T)^2$

$= l_0^2 (1 + 2\alpha \Delta T + \alpha^2 \Delta T^2)$

neglect

$= l_0^2 (1 + 2\alpha \Delta T)$

3 a) $Q = 0$

$$\gamma = \frac{C_p}{C_v} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3}$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} V_1 = \left(\frac{P_1}{P_2}\right)^{\frac{3}{5}} V_1$$

$$T_1 = \frac{P_1 V_1}{nR}$$

$$T_2 = \frac{P_2 V_2}{nR} = \frac{P_2 \left(\frac{P_1}{P_2}\right)^{\frac{3}{5}} V_1}{nR}$$

$$\Delta E = \frac{3}{2} nR\Delta T = \frac{3}{2} nR\left(\frac{P_2\left(\frac{P_1}{P_2}\right)^{\frac{3}{5}} V_1}{nR} - \frac{P_1 V_1}{nR} \right)$$

$$\Delta E = \frac{3}{2}\left(P_2 \left(\frac{P_1}{P_2}\right)^{\frac{3}{5}} V_1 - P_1 V_1 \right)$$

$$\Delta E = Q - W \Rightarrow W = -\Delta E$$

$$\boxed{W = \frac{3}{2}\left(P_1 V_1 - P_2 \left(\frac{P_1}{P_2}\right)^{\frac{3}{5}} V_1 \right)}$$

b) $\Delta T = 0$

$$\Delta E = \frac{3}{2} nR\Delta T = 0$$

$$\Delta E = Q - W = 0 \Rightarrow Q = W$$

$$W = \int_{V_2}^{V_3} P\,dV = nRT_2 \int_{V_2}^{V_3} \frac{dV}{V} = nRT_2 \ln\left(\frac{V_3}{V_2}\right)$$

$PV = nRT$ T is constant

$$\frac{PV}{nR} = \text{constant} \Rightarrow P_2V_2 = P_3V_3$$

$$\frac{P_2}{P_3} = \frac{V_3}{V_2}$$

$$W = nRT_2 \ln\left(\frac{P_2}{P_3}\right)$$

$$P_3 = P_1$$

$$T_2 = \frac{P_2\left(\frac{P_1}{P_2}\right)^{\frac{3}{5}} V_1}{nR}$$ from part a

$$\boxed{\begin{aligned} W &= P_2\left(\frac{P_1}{P_2}\right)^{\frac{3}{5}} V_1 \ln\left(\frac{P_2}{P_1}\right) \\ Q &= W \end{aligned}}$$

c)

$$\Delta E = \frac{3}{2}nR(T_1 - T_3) = \frac{3}{2}nR(T_1 - T_2)$$

$$\Delta E = \frac{3}{2}nR\left(\frac{P_1V_1}{nR} - \frac{P_2\left(\frac{P_1}{P_2}\right)^{\frac{3}{5}}V_1}{nR}\right)$$

$$\Delta E = \frac{3}{2}\left(P_1V_1 - P_2\left(\frac{P_1}{P_2}\right)^{\frac{3}{5}}V_1\right)$$

$$Q = nC_p\Delta T = \frac{5}{2}nR(T_1 - T_2)$$

$$\boxed{Q = \frac{5}{2}\left(P_1V_1 - P_2\left(\frac{P_1}{P_2}\right)^{\frac{3}{5}}V_1\right)}$$

$$\Delta E = Q - W \Rightarrow W = Q - \Delta E$$

$$W = \left(\frac{5}{2} - \frac{3}{2}\right)\left(P_1V_1 - P_2\left(\frac{P_1}{P_2}\right)^{\frac{3}{5}}V_1\right)$$

$$\boxed{W = P_1V_1 - P_2\left(\frac{P_1}{P_2}\right)^{\frac{3}{5}}V_1}$$

d)

$$PV = nRT$$

$$V = \frac{nRT}{P} \Rightarrow \frac{1}{V} = \frac{P}{nRT}$$

$$\frac{dV}{dT} = \frac{nR}{P}$$

$$\beta = \frac{1}{V}\frac{dV}{dT}$$

$$\beta = \left(\frac{P}{nRT}\right)\left(\frac{nR}{P}\right)$$

$$\boxed{\beta = \frac{1}{T}}$$

4) (35 pts.) One mole of water is cooled from T_1=25 °C to T_2=0 °C and frozen in a refrigerator. You can treat the heat being taken out of the water as the low-temperature heat input for the refrigerator. The refrigerator operates at maximum theoretical efficiency (no entropy created) and exhausts heat into a second mole of water at (again) T_1=25 °C, heating it to T_3=100 °C and converting a fraction (n' mole) into vapor.

Note: For parts *a*) through *d*), express your answer for each part in terms of any or all of the quantities given in the problem, T_1, T_2, T_3 and n', and relevant physical constants. You can use m as the mass of one mole of water. Also remember that there are two thermodynamic processes for each of these parts: the cooling and the freezing, or, the warming and the vaporization.

a) Find the total amount of heat, $|Q_l|$, that flows out of the first mole of water.

·Assume all the water gets frozen

$$Q_{l_{25°c \to 0°c}} = mc(T_2 - T_1)$$

$$Q_{l_{Liquid \to Solid}} = mL_f$$

$$|Q_l| = |Q_{l_{25 \to 0°c}} + Q_{l_{Liquid \to solid}}|$$

Answer:

$$\boxed{|mc(T_2 - T_1) - mL_f|}$$

b) Find the total amount of entropy, $|\Delta S_l|$, that flows out of the first mole of water.

$$\Delta S_{l_{25^\circ \to 0^\circ C}} = \int_{25^\circ C}^{0^\circ C} \frac{1}{T}\, dQ$$

$$= \int_{25^\circ C}^{0^\circ C} \frac{1}{T}\, mc\, dT$$

$$= mc \ln\left(\frac{T_2}{T_1}\right)$$

numerically convert to Kelvin! $T = 273 + T_n$

$$\Delta S_{l_{\text{liquid}\to\text{solid}}} = \frac{\Delta Q_{\text{liquid}\to\text{solid}}}{T_2}$$

$$= \frac{mL_f}{T_2}$$

<u>Answer:</u>

$$\boxed{\left| mc \ln\left(\frac{T_2}{T_1}\right) - m\frac{L_f}{T_2} \right|}$$

c) Find the total amount of heat, $|Q_2|$, that flows into the second mole of water.

$$Q_{2_{25°C \to 100°C}} = mc\,(T_3 - T_2)$$

$$Q_{2\,\text{vaporize}} = n' m\, L_v$$

Answer:

$$\boxed{\left| mc(T_3 - T_2) + n' m\, L_v \right|}$$

d) Find the total amount of entropy, $|\Delta S_2|$, that flows into the second mole of water.

$$\Delta S_{25^\circ \to 100^\circ} = \int_{25^\circ}^{100^\circ} \frac{1}{T}\, dQ$$

$$= mc \int_{T_1}^{T_3} \frac{1}{T}\, dT$$

$$= mc \ln\left(\frac{T_3}{T_1}\right)$$

$$\Delta S_{\text{liquid} \to \text{gas}} = \frac{\Delta Q}{T_3}$$

$$= \frac{n' m L_v}{T_3}$$

Answer:

$$\boxed{\left| mc \ln\left(\frac{T_3}{T_1}\right) + \frac{n' m L_v}{T_3} \right|}$$

e) What is $\Delta S_1 + \Delta S_2$? Note that there are no absolute value signs around the entropy changes here. (*Hint*: Can this process be reversed?)

$$mc\ln\left(\frac{T_2}{T_1}\right) - m\frac{L_f}{T_2} + mc\ln\left(\frac{T_3}{T_1}\right) + \frac{n' m L_v}{T_3}$$

$$= mc\ln\left(\frac{T_2 T_3}{T_1^2}\right) - m\left(\frac{L_f}{T_2} - \frac{n' L_v}{T_3}\right)$$

Answer:

0 reversible

f) Find n'.

$$mc\ln\left(\frac{T_2T_3}{T_1^2}\right) - m\left(\frac{L_f}{T_2} - \frac{n'L_v}{T_3}\right) = 0$$

$$c\ln\left(\frac{T_2T_3}{T_1^2}\right) = \frac{L_f}{T_2} - \frac{n'L_v}{T_3}$$

$$n' = \frac{T_3}{L_v}\left[\frac{L_f}{T_2} - c\ln\left(\frac{T_2T_3}{T_1^2}\right)\right]$$

Formulaic Answer:

$$\boxed{\frac{T_3}{L_v}\left[\frac{L_f}{T_2} - c\ln\left(\frac{T_2T_3}{T_1^2}\right)\right]}$$

$T_1 = 298K \quad T_2 = 273K \quad T_3 = 373K$

$L_f = 3.33\times10^5 \frac{J}{kg} \quad L_v = 22.6\times10^5 \frac{J}{kg}$

$C = 4.19\times10^3 \frac{J}{kg\,C}$

Numerical Answer:

$$\boxed{n' = 0.107}$$

f) How much work, $|W|$, must be done by the refrigerator?

$$|Q_2| = |W| + |Q_1|$$

$$|W| = |mc(T_3 - T_2) + n'mL_v| - |mc(T_1 - T_2) - mL_f|$$

$$= |mc(T_3 - T_2) + n'mL_v - mc(T_1 - T_2) - mL_f|$$

Formulaic Answer:

$$\boxed{m[c(T_3 - T_2) + n'mL_v - m(T_1 - T_2) - mL_f|}$$

Numerical Answer:

$$\boxed{W = 2116\ J}$$