UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
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Structural Engineering,
Mechanics and Materials
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1. The relevant boundary conditions are $\varphi(0) = 0$ and $GJ\varphi'(L) = 0$.

$$GJ\varphi'' = -t_o H(z-a) \tag{1}$$
$$GJ\varphi' = -t_o\langle z-a\rangle + C \tag{2}$$
$$GJ\varphi = -t_o\langle z-a\rangle^2/2 + Cz + D \tag{3}$$

Solving for $C, D$ gives $D = 0$ and $C = t_o(L-a)$, to yield

$$\underline{\underline{\varphi(z) = \frac{t_o}{GJ}\left[-\langle z-a\rangle^2/2 + (L-a)z\right]}} \tag{4}$$

Evaluating at $z = a/2$ gives:

$$\underline{\underline{\varphi(z) = \frac{t_o}{GJ}(L-a)a/2}} \tag{5}$$

2. Note that $R(x) = P$, where $P$ is to be determined, and the strain is homogeneous.

$$P = \int_A \sigma \, dA \tag{6}$$
$$= \int_{A_1} E_1\varepsilon^3 \, dA + \int_{A_2} E_2\varepsilon \, dA \tag{7}$$
$$= A_1E_1\varepsilon^3 + A_2E_2\varepsilon \tag{8}$$
$$= \underline{\underline{A_1E_1\left(\frac{\Delta}{L}\right)^3 + A_2E_2\left(\frac{\Delta}{L}\right)}} \tag{9}$$

where $A_1 = \pi R_1^2$ and $A_2 = \pi(R_2^2 - R_1^2)$.

3. In the final configuration the ring is in a state of strain with

$$\varepsilon_{\theta\theta} = \frac{D_1 - D_2}{D_2} \approx \frac{\sigma_{\theta\theta}}{E} - \frac{\nu}{E}\left(\underbrace{\sigma_{zz}}_{=0} + \underbrace{\sigma_{rr}}_{\approx 0}\right) \tag{10}$$

This gives $\sigma_{\theta\theta} = E(D_1 - D_2)/D_2$. Force balance on the ring now gives:

$$ptD_1 = 2\sigma_{\theta\theta}t^2 \tag{11}$$
$$p = 2\sigma_{\theta\theta}t/D_1 \tag{12}$$
$$p = \underline{\underline{2Et\frac{D_1 - D_2}{D_1D_2}}} \tag{13}$$