MIDTERM 2 Fall-2015

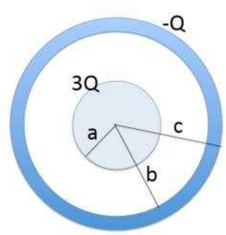
Instructor: Prof. A. LANZARA

TOTAL POINTS: 90

Show all work, and take particular care to explain what you are doing. Partial credit is given. Please use the symbols described in the problems, define any new symbol that you introduce and label any drawings that you make. If you get stuck, skip to the next problem and return to the difficult section later in the exam period.
All answers should be in terms of variables.

PROBLEM 1 (total 20pts)
A solid insulating sphere of radius a carries a net positive charge 3Q, uniformly distributed throughout its volume. Concentric with this sphere is a conducting spherical shell with inner radius b and outer radius c. The outer shell carries a charge of –Q as shown in the figure below.
Find the electric field (magnitude and direction) at locations:

a) (4pts) within the sphere ($r<a$);
b) (4pts) between the sphere and the shell ($a<r<b$)
c) (4pts) inside the shell ($b<r<c$)
d) (4pts) outside the shell ($r>c$)
e) (4pts) What charges appear on the inner and outer surfaces of the shell?

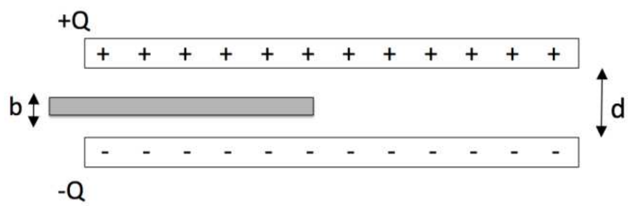


PROBLEM 2 (Points 20)

A slab of copper of thickness b length L/2 and width W is thrust into a parallel-plate capacitor of length L and width W, as shown below. The plates carry total charge +Q and –Q. The capacitor is isolated from the environment.

a) (10pts) What is the capacitance after the slab is introduced? Do not simply state the capacitance of a parallel plate capacitor, but show how you get it.

b) (10pts) How much work is done on the slab as it is inserted? Is the slab pulled in or do you have to push it in? Explain.



PROBLEM 3 (total 20pts)

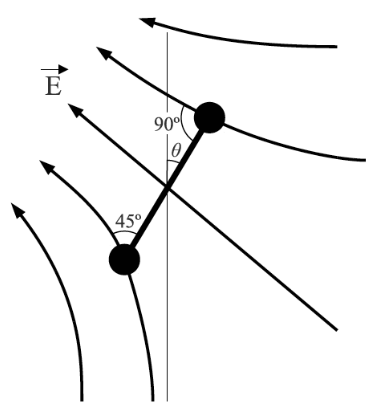
Consider an electric dipole in a non-uniform electric field as shown in the figure below. The dipole consists of two charges +Q and –Q separated by a distance d. The electric field is in the same plane as the dipole and the dipole is rotated by an angle θ from the vertical axis. The electric field at the positive charge has magnitude E_+ and is at an angle of 90^o to the axis of the dipole. The electric field at the negative charge has magnitude E_- and is at an angle of 45^o to the axis of the dipole.

a) (5 points) Is there a torque about the center of the dipole? Explain your reasoning. If there is a torque, find its direction and magnitude.

b) (5 points) What is the potential due to the dipole at the center of the dipole, if the potential for the dipole is zero at infinity?

c) (5 points) Draw the dipole orientation when its potential energy is at its maximum and at its minimum. Assume that the dipole can only rotate, not translate.

d) (5 points) Is there a net force on the dipole? Explain your reasoning.

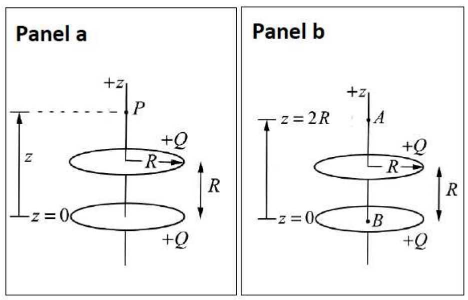


PROBLEM 4 (total 20pts)

Two rings with radius R have equal charge +Q uniformly distributed around each of them. The rings are parallel and separated by a distance R. Let z be the vertical coordinate, with z = 0 taken to be the center of the lower ring.

a) (10points) What is the potential difference, $V(P)-V(\infty)$, between a point P located on the vertical axis a distance +z from the center of the lower ring, and infinity (panel a)? Set $V(\infty) = 0$.

b) (10points) An electron of mass m and charge -e is released from rest at point A on the vertical axis, a distance z = 2R from the center of the lower ring. What is the speed of the electron when it reaches the point B at the center of the lower ring (panel b)?



PROBLEM 5 (total 20pts)

A remote control contains four AAA batteries. Two batteries are in series (called them a pair), and the two pairs are connected in parallel. Each AAA battery provides an EMF=E and has an internal resistance of R_o. When a button is pressed, the battery combination is connected across a light bulb with an equivalent resistance $R_{bulb} = 80\ R_o$.

a) (4 points) Draw the equivalent circuit diagram of the full system, label the EMFs, internal resistances, and the light bulb resistance.

b) (4 points) Draw the reduced equivalent circuit for a single battery with internal resistance, and the light bulb resistance. Label the EMF and resistances with their values.

c) (4 points) Draw the reduced equivalent circuit with a single EMF and one resistance. Again, label the EMF and resistance with their values.

d) (4 points) What is the current through each battery?

e) (4 points) How much power does the EMF in each battery provide?

$$y(t) = \frac{B}{A}(1 - e^{-At}) + y(0)e^{-At}$$
$$\text{solves } \frac{dy}{dt} = -Ay + B$$

$$y(t) = y_{max}\cos(\sqrt{A}t + \delta)$$
$$\text{solves } \frac{d^2y}{dt^2} = -Ay$$

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} \\ d\vec{l} &= dx\hat{x} + dy\hat{y} + dz\hat{z} \\ & \text{(Cartesian Coordinates)} \end{aligned}$$

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial f}{\partial z}\hat{z} \\ d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + dz\hat{z} \\ & \text{(Cylindrical Coordinates)} \end{aligned}$$

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial f}{\partial \phi}\hat{\phi} \\ d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + r\sin(\theta)d\phi\hat{\phi} \\ & \text{(Spherical Coordinates)} \end{aligned}$$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n! 2^{2n+1}}\sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int (1+x^2)^{-1/2} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{x}{\sqrt{1+x^2}} dx = \sqrt{1+x^2}$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha - 1)\alpha}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$