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**ME 40 Thermodynamics**
**Fall 2015, Midterm 2**

Name: [redacted]

1. Carnot developed the cycle comprised of four processes shown in the figures below. reversible

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a. Use the boxes to describe the **key** descriptors of the processes and draw the process on the TS diagram. (hint: Processes 1-2 and 3-4 are isentropic)

b. Start with the equation $\delta q = C_v dT + \frac{RT}{v} dv$. Apply it to the four processes in the Carnot cycle to show that $\left(\frac{q_H}{q_L}\right)_{rev} = \frac{T_H}{T_L}$.

1→2 | Isentropic and adiabatic, so $ds = dq = 0 = C_v dT + \frac{RT}{V} dV$

$\Rightarrow \int_{T_1}^{T_2} C_v dT = \int_{V_1}^{V_2} \frac{RT}{V} dV$

$\Rightarrow -C_v(T_2 - T_1) = RT \ln\left(\frac{V_2}{V_1}\right)$ ← where $T_2 = T_3 = T_H$ and $T_1 = T_4 = T_L$

2→3 | Heat addition, constant temp, so $dq = C_v(0) + \frac{RT}{V} dV$

$\Rightarrow Q_{2\to3} = Q_H = \int_{V_2}^{V_3} \frac{RT}{V} dV$

$\Rightarrow Q_H = RT \ln\left(\frac{V_3}{V_2}\right)$

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3→4 | same as 1→2:

$\Rightarrow -C_v(T_4 - T_3) = R \ln\left(\frac{V_4}{V_3}\right)$

~~$\Rightarrow -C_v(T_H - T_L) = RT \ln\left(\frac{V_2}{V_1}\right)$~~

4→1 | same as 2→3:

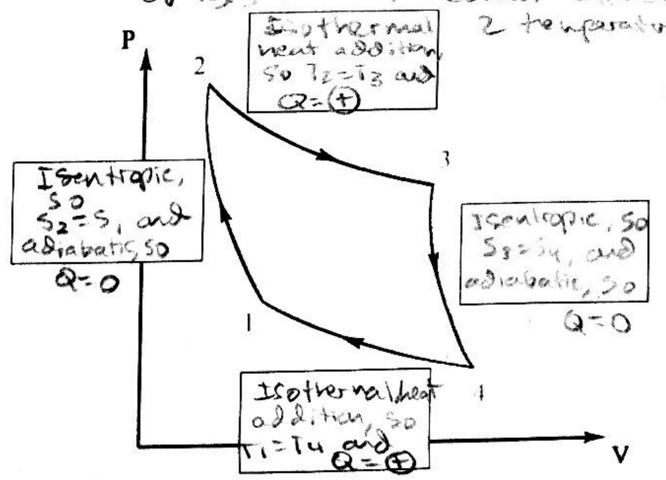
$\Rightarrow Q_L = RT \ln\left(\frac{V_1}{V_2}\right)$

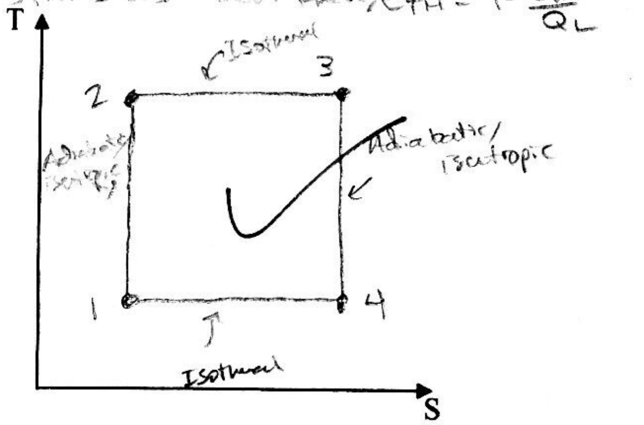
~~$\Rightarrow \frac{Q_H}{Q_L} = \frac{RT \ln\left(\frac{V_3}{V_2}\right)}{RT \ln\left(\frac{V_1}{V_2}\right)} = \frac{RT}{C_v} \ln\left(\frac{V_2}{V_1}\right) + T_L$~~

c. Comment on the difference between using q and T in calculating thermal efficiency.

Using T for thermal efficiency only applies when heat addition occurs isothermally so that $\frac{q_H}{q_L} = \frac{T_H}{T_L}$. This is the maximum efficiency one can ever have between those 2 temperature reservoirs, and occurs in the Carnot cycle. Q's yield the maximum efficiency of that cycle, which must be equal or less than the carnot efficiency corresponding to between those 2 temperature states as well. Here, $e_{TH} = 1 - \frac{Q_H}{Q_L}$

one would use $e_{TH} = 1 - \frac{T_H}{T_L}$



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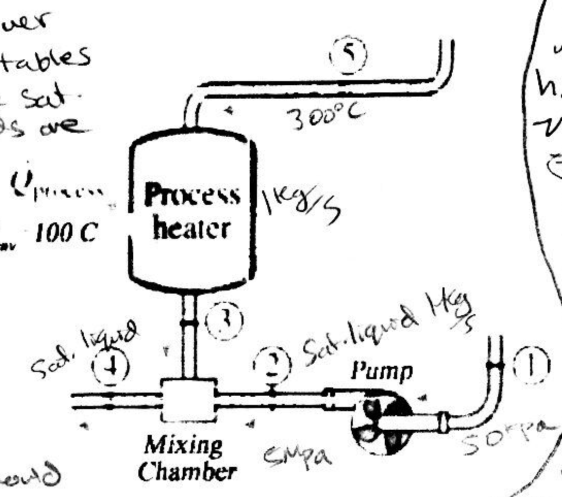
2. Focus on one small part of a cogeneration vapor power cycle system, shown in the device-pipe diagram below:

The pump operates between 50 kPa and 5 MPa with an isentropic efficiency of 80%. The inlet of the process heater (state 5) is at 300 C. Both the inlet of the pump (state 1) and the outlet of the ideal mixing chamber (state 4) is at saturated liquid state. The mass flow rate through both the pump and the process heater is 1 kg/s.

   a. Fill in the missing properties of each state in the below table. (Please show how you get all the values to receive credit)
   b. Show the processes on the T-s diagram. (Label all the states and connect them with lines representing the correct processes)
   c. What is the rate of process heat generation?
   d. What is the rate of entropy generation of this system?

This is a vapor power cycle. We will use the tables for the values, and assume sat. liquids or compressed liquids are incompressible.




$1 \to 2$ | This is a pump whose effeciency is 80% (isentropic). This means

$e_p = 0.8 = \frac{h_{2s} - h_1}{h_2 - h_1}$, where

$h_{2s}$ is the enthalpy that would be at state 2 if the pump was isentropic. State 1 is given to be sat. liquid at $P_1 = 50kPa$, so $\Rightarrow h_1 = h_f = 340.54 \frac{kJ}{kg}$

To get $h_{2s}$ we calculate it using $S_2 = S_1$, where $S_1 = S_f$ at $P_1 = 50kPa \Rightarrow S_1 = 1.0912$. Knowing $P_2 = 5000kPa$, we use compressed table interpolation:

$h_{2s} = \left(\frac{1.0912 - 1.0723}{1.1034 - 1.0723}\right)(422.85 - 338.96) + 338.96$ ~~$v_1(P_2 - P_1)$~~

$\Rightarrow h_{2s} = 345.821 \frac{kJ}{kg}$

Use that it's effeciency is 80%

$\Rightarrow 0.8 = \frac{345.821 - 340.54}{h_2 - 340.54}$

$\Rightarrow h_2 = 347.14095 \frac{kJ}{kg}$

$5 \to 3$ | We know state 5, it is 300°C at pressure of 5000kPa (Process heater acts like a pipe) $\Rightarrow h_5 = 2,925.7$

The process heater removes heat, keeping the pressure constant

$\dot{Q}_{process} = \dot{H}_5 - \dot{H}_3 = \dot{m}(h_5 - h_3)$

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| State | P [kPa] | h [kJ/kg] |
|---|---|---|
| 1 | 50 | 340.54 |
| 2 | 5000 | 347.14095 |
| 3 | 5000 | 1,961.8591 |
| 4 | 5000 | 1,154.5 |
| 5 | 5000 | 2,925.7 |

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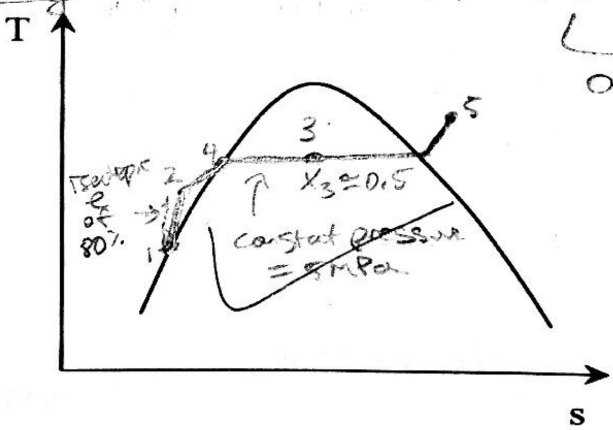

$\underline{2+3 \Rightarrow 4}$ | This is an ideal mixer, so:

$$\dot{E}_{in} = \dot{E}_{out}$$

All at constant pressures, such that $P_2 = P_3 = P_4 = 5000\,kPa$.

We know the outlet at state 4 is a sat. liquid, so

$h_4 = h_f \text{ @ } P = 5000\,kPa \Rightarrow h_4 = 1{,}154.5\,\frac{kJ}{kg}$

Making energy balance, we have:

$$H_2 + H_3 = H_4$$ ← mass flows at 2 and 3 are $1\,\frac{kg}{s}$, while at 4 it's $1+1=2\,\frac{kg}{s}$

$$\Rightarrow h_2 + h_3 = 2h_4$$

$$\Rightarrow h_3 = h_4 - h_2 = 2{,}309 - 347.14095$$

$$\Rightarrow h_3 = 1{,}961.8591\,\frac{kJ}{kg}$$

b). All the states of 2, 3, 4 and 5 should be on a constant pressure line, where 2 is compressed liquid, 4 is sat. liquid, 5 is superheated steam, and $h_3 = 807.35905 \text{ @ } P_3 = 5000\,kPa$

$\Rightarrow x_3 = 0.49238$ = sat. mixture.

State 1 is also sat.

c). Process heat gen is given by:

$$\dot{Q}_{process} = H_5 - H_3 = \dot{m}_1(h_5 - h_3)$$

$$= 1(2{,}925.7 - 1{,}961.851)$$

$$\Rightarrow \dot{Q}_{process} = 963.849\,\frac{kJ}{s}$$

$$\Rightarrow \boxed{\dot{Q}_{process} = 963.849\,kW}$$

d). $\Delta S_{sys} = S_{in} - S_{out} + S_{gen}$

We are taking the cycle machine to be the system, which is in steady state, so $\Delta S_{sys} = 0$, and there is no heat coming in here, so $S_{in} = 0$:

$$\Rightarrow 0 = -S_{out} + S_{gen}$$

$\Rightarrow S_{gen} = S_{out}$, where the entropy leaving the system is given by $S_{out} = \frac{Q}{T} = \frac{\dot{Q}_{process}}{T_{env}}$

$$\Rightarrow S_{out} = \frac{963.849\,kW}{(273+100)\,°K}$$ Sin, Sout?

$$\Rightarrow S_{out} = 2.584045\,\frac{kW}{°K}$$ −10

$$\Rightarrow \boxed{S_{gen} = 2.584045\,\frac{kW}{°K}}$$