Physics 7B

Midterm 2: Monday November 2nd, 2015

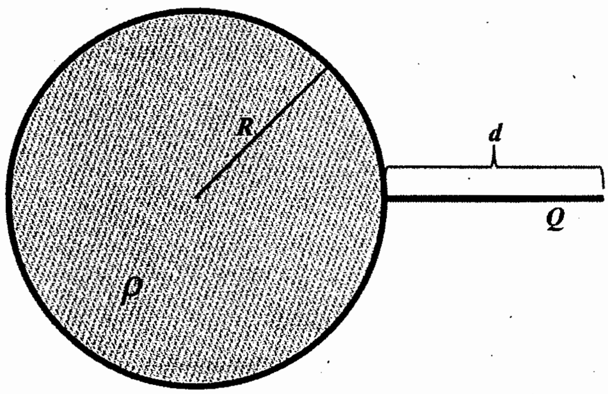
Instructors: Prof. R.J. Birgeneau/Dr. A. Frano

Total points: 100 (6 problems)

Show all your work and take particular care to explain what you are doing. Partial credit can be given. Please use the symbols described in the problems or define any new symbol that you introduce. Label any drawings that you make. ***Good luck!***

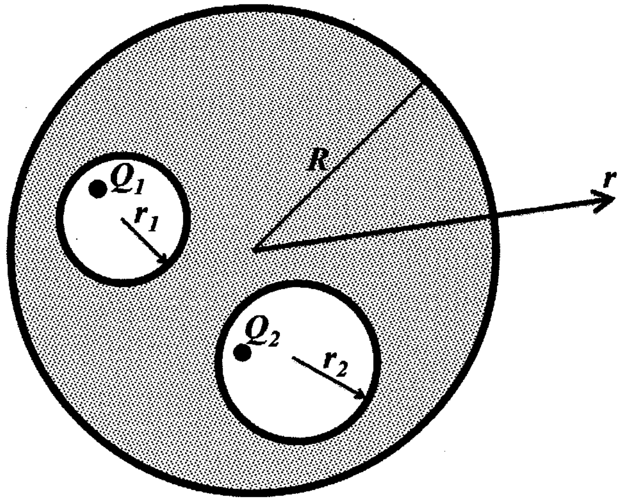
Problem 1 (15 pts)

A uniformly charged solid sphere of radius R carrying volume charge density ρ is centered at the origin. Find the force (magnitude and direction) on a uniform line charge that has a total charge Q. The line is oriented radially with respect to the sphere with its ends at R and $R + d$.



Problem 2 (15 pts)

An uncharged solid conducting sphere of radius R, centered at the origin, contains two spherical cavities of radii r_1 and r_2, respectively. Point charge Q_1 in then placed within the cavity of radius r_1, and point charge Q_2 in then placed within the cavity of radius r_2, as shown below. Determine the resulting electric field vector for $r > R$, where r is the distance from the origin.

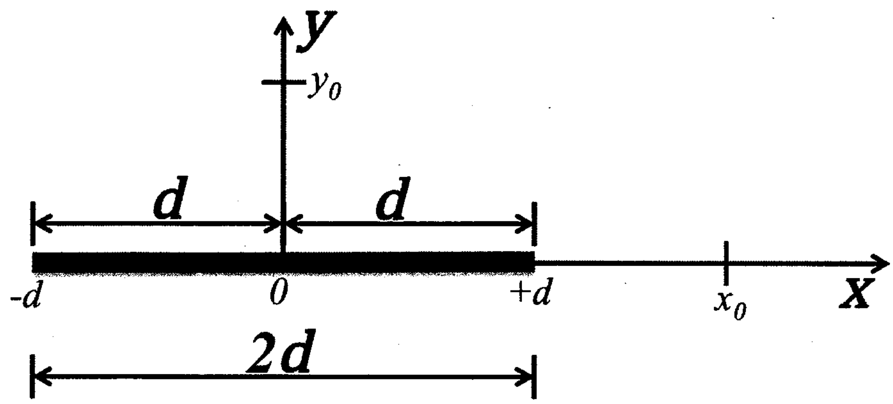


Problem 3 (15 pts)

Consider a thin rod of length $2d$ centered on the x-axis as shown below. The rod has a nonuniform linear charge distribution $\lambda = ax$. Determine the potential V for

(a) (5 pts) a point along the y-axis at a distance y_0 from the origin

(b) (10 pts) points along the x-axis outside the rod, at a distance x_0 from the origin



Problem 4 (15 pts)

The plates of a parallel-plate capacitro have area A, separation x, and are connected to a battery with voltage V. While connected to the battery, the plates are pulled apart until they are separated by $2x$.

(a) (5 pts) What are the initial and final energies stored in the capacitor?

(b) (5 pts) How much work is required to pull the plates apart (assume constant speed)?

(c) (5 pts) How much energy is exchanged with the battery?

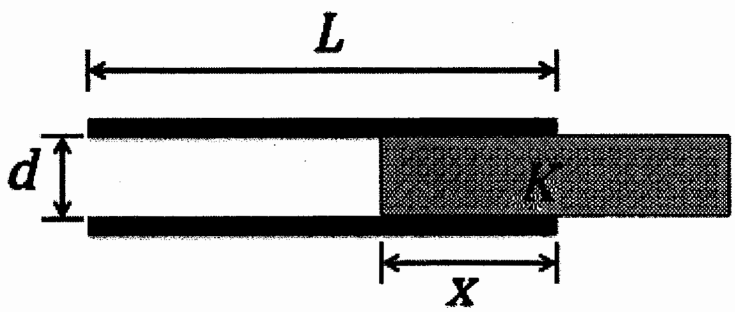
Problem 5 (20 pts)

A slab of width d and dielectric constant K is inserted a distance x into the space between the square parallel plates (of side L) of a capacitor as shown below. Determine the following, all as function of x:

(a) (5 pts) the capacitance,

(b) (5 pts) the energy stored if the potential difference is constant V_0,

(c) (5 pts) the magnitude and direction of the force exerted on the slab.



Problem 6 (20 pts)

Suppose that we are building a rechargeable appliance which will be powered by two capacitors, each with capacitance C. While the device is charging, the capacitive power supply is placed in a circuit with a battery of potential difference V. There are two possible ways to charge the capacitors, in series or in parallel, as shown below (Part 1).

Part 1:

(a) (5 pts) Which configuration will store the most charge? Explain why.

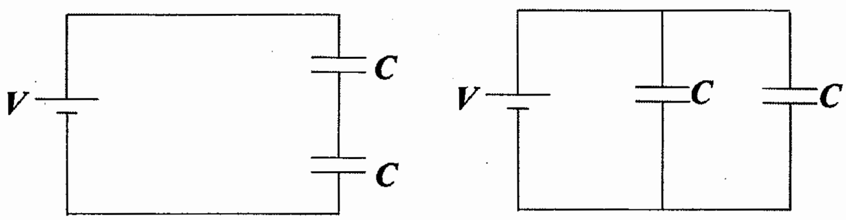
Part 2: For this part of the problem, assume the capacitor configuration from Part 1 that held the most charge. Once the power supply is charged, it is taken out of the circuit with the battery and placed in the circuit shown below. Every resistor in the circuit has a resistance of R.

(b) (5 pts) Calculate the equivalent resistances of branches 1 and 2.

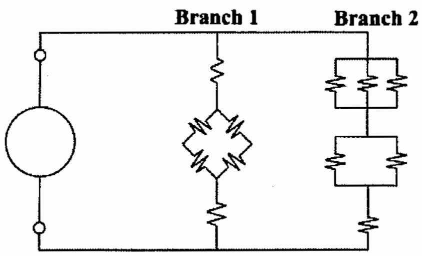
(c) (5 pts) Find the ratio of current through branch 1 to the current in branch 2.

(d) (5 pts) Write a function describing the charge on the capacitor plates as a function of time.

Part 1



Part 2



$$y(t) = \frac{B}{A}(1 - e^{-At}) + y(0)e^{-At}$$
$$\text{solves } \frac{dy}{dt} = -Ay + B$$

$$y(t) = y_{max}\cos(\sqrt{A}t + \delta)$$
$$\text{solves } \frac{d^2y}{dt^2} = -Ay$$

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} \\ d\vec{l} &= dx\hat{x} + dy\hat{y} + dz\hat{z} \end{aligned}$$
(Cartesian Coordinates)

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial f}{\partial z}\hat{z} \\ d\vec{l} &= dr\hat{r} + r d\theta\hat{\theta} + dz\hat{z} \end{aligned}$$
(Cylindrical Coordinates)

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial f}{\partial \phi}\hat{\phi} \\ d\vec{l} &= dr\hat{r} + r d\theta\hat{\theta} + r\sin(\theta)d\phi\hat{\phi} \end{aligned}$$
(Spherical Coordinates)

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n! 2^{2n+1}} \sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int (1+x^2)^{-1/2} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{x}{\sqrt{1+x^2}} dx = \sqrt{1+x^2}$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha-1)\alpha}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$