Sample MT # 1

Total Spin of lhs: $S = 1$ } $j = 1$ for lhs
Total ang. mom. of lhs : $l = 0$ }

j is conserved

neutron has spin $\frac{1}{2}$: Thus rhs $S = 0, 1$

But $S = 0$ is ruled out :

If $S = 0$ (antisymmetric), we would require a symmetric orbital state, i.e. even value of l. This would require an even value of j, which would not conserve j.

Thus $S = 1$ (symmetric)

this requires an antisymmetric orbital state, i.e. $l = 1, 3 \ldots$

But since $j = 1$, $s = 1$, the only allowed values is $j = 1$

Thus $l = 1$, $s = 1$, $j = 1$

(a) For the symmetric case, we can have wavefunctions of the form

$$\psi_i(1)\psi_i(2)$$

or, for $j \neq i$,

$$\frac{1}{\sqrt{2}}\left(\psi_i(1)\psi_j(2) + \psi_j(1)\psi_i(2)\right)$$

For the first case, there are n possible states ($n = 1, 2, ...n$). For the second case, i can take on n different values, and since $j \neq i$, j can take on $n-1$ values, for a total of $n(n-1)$ states. This double counts, however, since ($i = a, j = b$) is equivalent to ($i = b, j = a$). So the contribution due to $i \neq j$ terms is $n(n-1)/2$ [this is the same as n choose 2]. The total number of symmetric states is then,

$$n + \frac{1}{2}n(n-1) = \boxed{\frac{1}{2}n(n+1).}$$

(b) The $i = j$ case is no longer allowed, since it is necessarily symmetric. For $i \neq j$, we can antisymmetrize:

$$\frac{1}{\sqrt{2}}\left(\psi_i(1)\psi_j(2) - \psi_j(1)\psi_i(2)\right)$$

So the number of states is just the number of $i \neq j$ states from part (a):

$$\boxed{\frac{1}{2}n(n-1).}$$

(c) The multiplicity of states for a particle with spin s is $n = 2s + 1$, so our expressions from above give that the ratio is

$$\frac{n(n+1)}{n(n-1)} = \frac{n+1}{n-1} = \frac{2s+2}{2s} = \boxed{\frac{s+1}{s}}$$

(a) Because the two electrons are in different orbital angular momentum states, they are inequivalent, so any of the two states accessible to the 2s electron may be paired with any of the six states available to the 2p electron, giving a degeneracy of $2 \times 6 = 12$. We are combining $l = 0$ with $l = 1$, which gives only $L = 1$. We are combining two spin-1/2 particles, so we can have $S = 0$ or $S = 1$. Again, the inequivalency removes any symmetrization requirements, so either combination is fine. The degeneracy of a given state is $2J + 1$. This gives

S	L	J	$^{2S+1}L_J$	Degeneracy
0	1	1	$^1P_1$	3
1	1	2, 1, 0	$^3P_{2,1,0}$	$5+3+1=9$

So the total degeneracy is $3 + 9 = 12$, which agrees with our earlier calculation.

(b) Because the two electrons have different values of n, they are inequivalent. They are each in an $l = 1, s = 1/2$ state, which has degeneracy of 6, so the total degeracy is $6 \times 6 = 36$. We are now combining two $l = 1$ states which gives us possible values for L of 2, 1, and 0. Again, the inequivalency of the electrons means that each of these may be paired with either spin state:

S	L	J	$^{2S+1}L_J$	Degeneracy
0	0	0	$^1S_0$	1
0	1	1	$^1P_1$	3
0	2	2	$^1D_2$	5
1	0	1	$^3S_1$	3
1	1	2, 1, 0	$^3P_{2,1,0}$	$5+3+1=9$
1	2	3, 2, 1	$^3D_{3,2,1}$	$7+5+3=15$

Summing these degeneracies, we get 36 as expected.

(c) There are only ten available states for an $l = 2, s = 1/2$ electron, and since we have ten electrons, there is only one possible configuration. Each spin up must be paired with a spin down, giving a total spin of $S = 0$. Similarly, we have total orbital angular momentum of $L = 0$. This gives us a term $^1S_0$, which has degeneracy 1.

(d) We now have 9 d-electrons, which is equivalent to a single hole. There are ten ways that we can put a spin-1/2 hole into an $l = 2$ state, so we expect degeneracy 10. Since there is only one hole, $S = s = 1/2$, and $L = l = 2$. Our possible terms are then $^2D_{\frac{5}{2},\frac{3}{2}}$, giving a degeneracy of $6 + 4 = 10$.

3. In the lectures we derived the anomalous Zeeman effect by writing

$$\vec{\mu}_{\text{atom}} = -g\frac{\mu_{\text{B}}}{\hbar}\vec{J}$$

where g is the Lande g factor. We neglected hyperfine interactions. One could account for them by further writing

$$\vec{\mu}_{\text{atom}} = -\frac{\mu_{\text{B}}}{\hbar}(g\vec{J} + g_{\text{I}}\frac{\mu_{\text{N}}}{\mu_{\text{B}}}\vec{I}) = -g_{\text{F}}\frac{\mu_{\text{B}}}{\hbar}\vec{F}.$$

Derive an expression for g_{F} in terms of g and g_{I}. Use the expression to calculate the weak field splitting of the H ground state. Draw a figure showing the energies with magnetic field strength and label with relevant quantum numbers.