Physics 7B

Midterm 1: Monday September 28th, 2015

Instructors: Prof. R.J. Birgeneau/Dr. A. Frano

Total points: 100 (5 problems)

Show all your work and take particular care to explain what you are doing. Partial credit can be given. Please use the symbols described in the problems or define any new symbol that you introduce. Label any drawings that you make. ***Good luck!***

Problem 1 (20 pts)

A thermally isolated container contains a mass M=2 kg of ice at temperature $T = 0°\text{C}$. An equal mass of water at temperature $T = 90°\text{C}$ is added to the container. The pressure P of the water-ice system is 1 atm.

Assume that the specific heat of the water does not depend on the temperature and is c_w=1 cal/(g K). The latent heat of fusion of ice is L=80 cal/g. Neglect the change of volume of ice and water and the specific heat of the container.

(a) (10 pts) What is the final equilibrium temperature?

(b) (10 pts) What is the approximate change in entropy of the water-ice system between the initial and final state? Use the approximation: $\ln(1 + x) \sim x - \frac{x^2}{2}$. State the answer in fraction form.

Problem 2 (20 pts)

The Otto cycle comprises four segments: (1) adiabatic expansion from $A \to B$, (2) isovolumetric cooling from $B \to C$, (3) adiabatic compression from $C \to D$, and (4) isovolumetric heating from $D \to A$.

(a) (2pts) Sketch the cycle in a PV diagram.

(b) (5pts) What is the heat absorbed by the gas in segments 1,2,3 and 4 in terms of P and V?

(c) (5pts) Calculate the efficiency $e = W/Q_H$ of this engine for an ideal diatomic gas, where W is the work done by the gas in a full cycle, and Q_H is the total heat flowing into the gas. Is it greater than, equal to, or less than the efficiency of the Carnot cycle? Show the comparison explicitly.

(d) (3pts) Would the efficiency increase or decrease for a monoatomic gas? Show the comparison explicitly.

(e) (5pts) Calculate the change in entropy across the entire cycle.

Problem 3 (20 pts)

Suppose N gas molecules each with mass m live on a plane, meaning they can only move in 2 dimensions. Furthermore, a square box of size l encloses the gas within its walls. The gas particles do not interact with each other, but when they reach the walls they elastically collide and change direction.

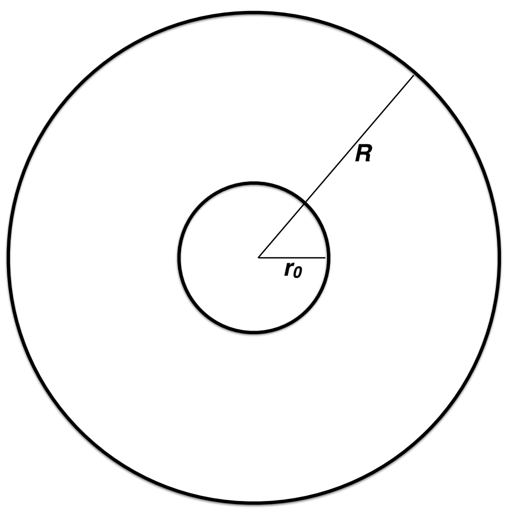
(a) (8 pts) If a molecule has velocity components $\vec{v} = \hat{x}v_x + \hat{y}v_y$ and $v_x = v_y$, how much time would go by between collisions on a given wall?

(b) (6 pts) How much force F needs to be applied at each wall to balance the force exerted by the gas molecules. (Assume all molecules behave like in (a).)

(c) (6 pts) Using the equipartition theorem's temperature definition, $T = mv^2/k_B$, derive the equation of state of the gas.

Problem 4 (20 pts)

Consider a monoatomic ideal gas, at pressure P, confined between a metal shell of radius R and a metal sphere of radius r_0. The gas and metal are initially at equilibrium at temperature T_0. The inner sphere metal has a volumetric expansion coefficient β. Ignore the thermal expansion of the outer shell (it has a very small expansion coefficient).



(a) (5 pts) The inner metal sphere is heated to a temperature T_f. Find the new radius, r_1 of the sphere in terms of the given variables.

(b) (5 pts) Assuming the expansion of the sphere to be instantaneous, find the final pressure of the gas in terms of the given variables and r_1.

(c) (5 pts) Find the temperature of the gas right after the compression.

(d) (5 pts) What is the root-mean-square (RMS) velocity of the gas after the compression? What would the RMS velocity be if the particles were only allowed to move radially?

Problem 5 (20 pts)

One mole of an ideal, diatomic gas is at pressure P_1, volume V_1, and temperature T_1. It then undergoes the following cycle: (A) it is heated at constant volume to point 2 at four times the initial pressure, (B) it expands adiabatically to point 3 at its original temperature, and (C) it compresses isothermally back to P_1.

(a) (10 pts) What are P_2, V_2, P_3, and V_3?

(b) (10 pts) What is ΔE, Q, and W for each step in the cycle? Assume there are no vibrational degrees of freedom. State your answers in terms of P_1, V_1, T_1, and R.