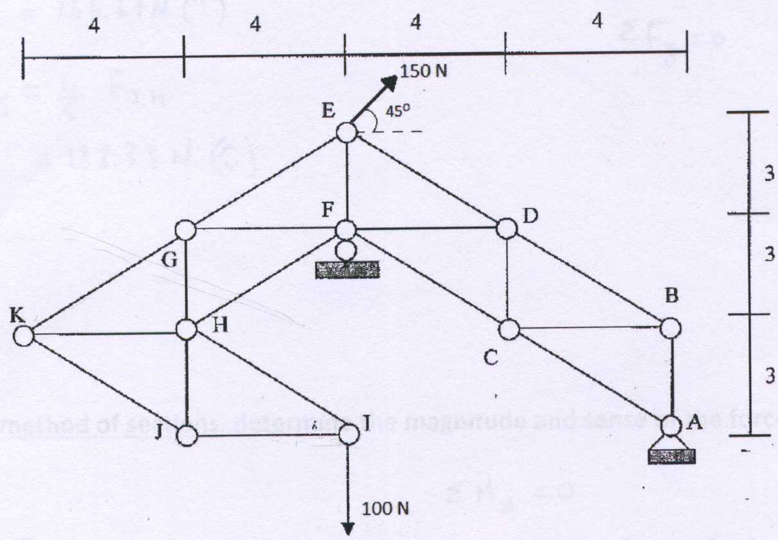
ME C85/CE C30 Mid-Term Exam #2, Spring 2013 *Name:* Abdulrahman Jbaily

Monday, April 8, 2013, 2–3 PM.

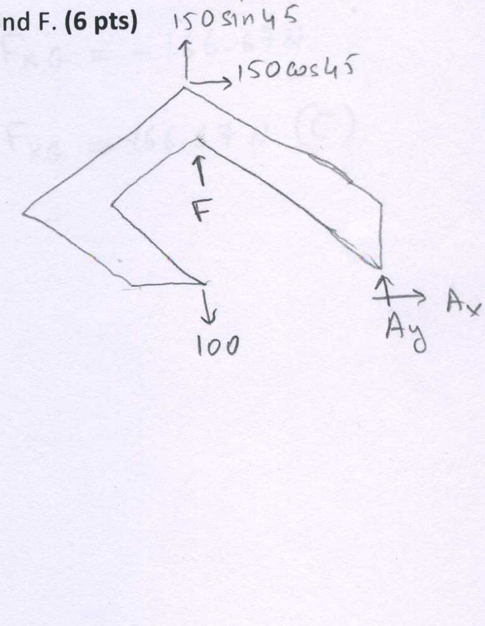
Please write your name at the top of each page as indicated and write all answers in the space provided. If you need additional space, write on the back sides. Do not remove or add any pages. **Assume all problems are two-dimensional unless noted otherwise. For all answers, where appropriate, provide units.** *Good luck!*

PROBLEM 1: 24 pts total

The truss shown below is supported by a hinge at A and a roller at F and is loaded by a vertical force of 100 N at I and a 150 N force at E making an angle of 45 degrees with the horizontal. Answer the following questions using clear free body diagrams and equilibrium equations:



a- Determine the magnitude and direction of the reaction forces at A and F. **(6 pts)**



$\Sigma F_x = 0 \Rightarrow A_x + 150\cos(45) = 0$

$A_x = -106.066\,N = 106.066\,N \leftarrow$

$+\circlearrowleft \; \Sigma M_F = 0 \Rightarrow -150\cos(45)(3) - A_x(6) + A_y(8) = 0$

$A_y = 119.32\,N \uparrow$

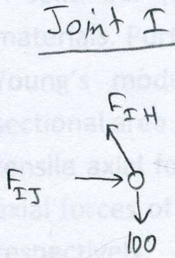
$\Sigma F_y = 0 \Rightarrow 150\sin(45) - 100 + 119.32 + F = 0$

$F = -125.386\,N$

$= 125.386\,N \downarrow$

b- Using the method of joints, determine the magnitude and sense (tension or compression) of the force in member JH. **(9 pts)**

Joint I



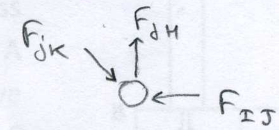
$\Sigma F_y = 0 \quad F_{IH}\left(\frac{3}{5}\right) = 100$

$F_{IH} = 166.67 N$ (T)

$\Sigma F_x = 0 \quad F_{IJ} = \frac{4}{5} F_{IH}$

$= 133.33 N$ (C)

Joint J



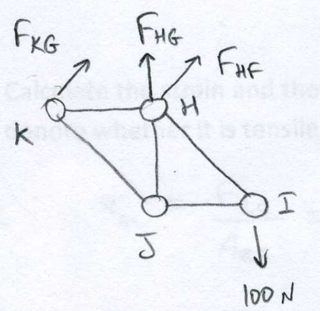
$\Sigma F_x = 0 \quad F_{JK}\left(\frac{4}{5}\right) = F_{IJ}$

$F_{JK} = 166.67 N$ (C)

$\Sigma F_y = 0 \quad F_{JH} = F_{JK}\left(\frac{3}{5}\right)$

$F_{JH} = 100 N$ (T)

c- Using the method of sections, determine the magnitude and sense of the force in member KG. **(9 pts)**



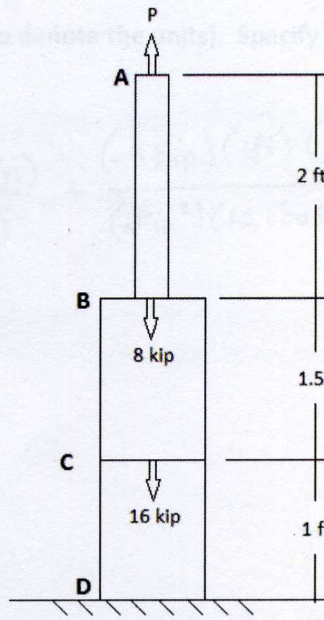
$\Sigma M_H = 0$

$F_{KG}\left(\frac{3}{5}\right)(4) + 100(4) = 0$

$F_{KG} = -166.67 N$

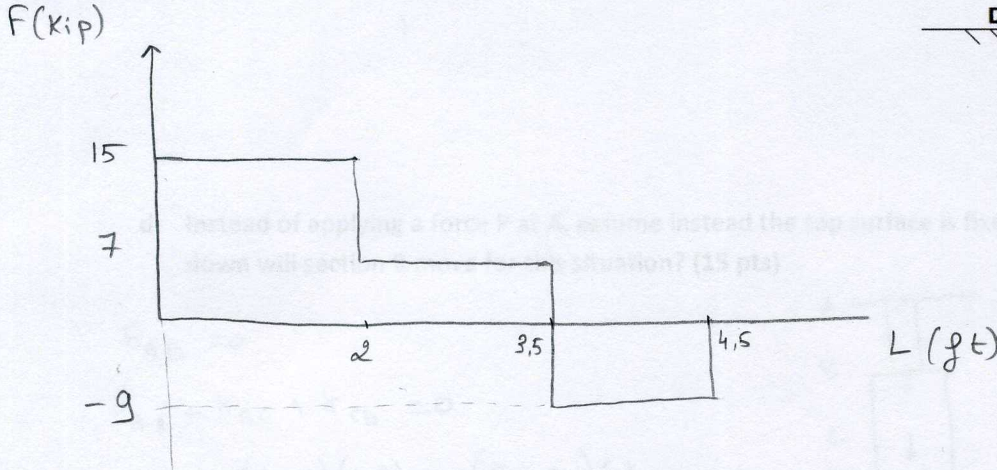
$F_{KG} = 166.67 N$ (C)

PROBLEM 2: 40 pts total



A solid bar (shown on the right) is made of two different materials. Portion AB has a cross sectional area A_{AB}=1 in^2 and a Young's modulus E_{AB}=29,000 ksi. Portion BD has a cross sectional area A_{BD}=2 in^2 and a Young's modulus E_{BD}=10,000 ksi. A tensile axial force P of 15 kip is applied at A, two compressive axial forces of 8 kip and 16 kip are applied at sections B and C respectively.

a- Plot a graph of the net axial force along the length of the bar. On your plot, let x=0 at point A, and denote all tensile forces with positive values. **(8 pts)**



b- Calculate the strain and the stress in both sections B-C and C-D. For each stress and strain, denote whether it is tensile or compressive. **(8 pts)**

B-C:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{7000\ \text{lb}}{2\ \text{in}^2} = 3500\ \text{psi}$$

$$\varepsilon_{BC} = \frac{\sigma_{BC}}{E_{BC}} = \frac{3500\ \text{psi}}{10{,}000{,}000\ \text{psi}} = 3{,}5 \times 10^{-4}$$

Tensile

C-D:

$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{-9000\ \text{lb}}{2\ \text{in}^2} = -4500\ \text{psi}$$

$$\varepsilon_{CD} = \frac{\sigma_{CD}}{E_{CD}} = \frac{-4500\ \text{psi}}{10{,}000{,}000\ \text{psi}} = -4{,}5 \times 10^{-4}$$

Compressive

c- Determine the vertical displacement of end A (and remember to denote the units). Specify if this displacement is upwards or downwards. **(9 pts)**

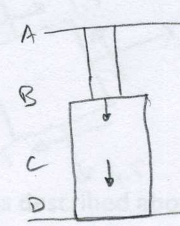
$$\delta_A = \delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD}$$

$$= \frac{(15\ kip)(2\ ft)(12\ in/ft)}{(1\ in^2)(29000\ ksi)} + \frac{(7\ kip)(1.5\ ft)(12\ in/ft)}{(2\ in^2)(10{,}000\ ksi)} + \frac{(-9\ kip)(1\ ft)(12\ in/ft)}{(2\ in^2)(10{,}000\ ksi)}$$

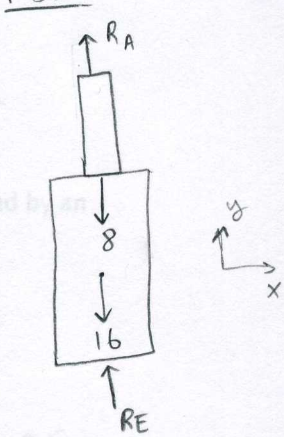
$$= 0.0124 + 6.3 \times 10^{-3} - 5.4 \times 10^{-3}$$

$$= 0.0133\ in \quad \text{upwards}$$

d- Instead of applying a force P at A, assume instead the top surface is fixed to a wall at A. How far down will section B move for this situation? **(15 pts)**



FBD



$$\delta_{AD} = 0$$

$$\delta_{AB} + \delta_{BC} + \delta_{CD} = 0$$

$$\frac{R_A(2)}{29000} + \frac{(R_A - 8)(1.5)}{2(10{,}000)} + \frac{(R_A - 24)(1)}{2(10{,}000)} = 0$$

$$\Rightarrow R_A = 9.28\ kip$$

from (*) $\Rightarrow R_E = 24 - R_A = 14.72\ kip$

$\Sigma F_y = 0$

(*) $R_A = 24 - R_E$

$\Rightarrow$ statically inditerminate

$$\delta_B = \delta_{AB} = \frac{R_A(2)}{29000} = \frac{9.28(2)}{29000} = 6.4 \times 10^{-4}\ ft \text{ down}$$

$$= 7.68 \times 10^{-3}\ in \text{ down}$$

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PROBLEM 3: 36 pts total

A thin-walled pressure vessel of 5-in. inner radius **r**, 0.25-in. wall thickness **t**, and with flat ends is fabricated from a 4-ft section of a pipe AB as shown below. The pressure inside the vessel is 300 psi. We will analyze stresses at a point on the vessel's external surface at coordinate y=0, z=r+t (for any value of x), as shown in Figure a.

a- When the vessel is just pressurized as described above, determine the stresses σ_x and σ_y and also the maximum shearing stress in the x-y plane and its orientation. Remember to indicate your units **(8 pts)**

$$\sigma_y = \frac{Pr}{t} = \frac{300(5)}{0.25} = 6000 \text{ psi} = 6 \text{ Ksi}$$

$$\sigma_x = \frac{1}{2}\sigma_y = 3000 \text{ psi} = 3 \text{Ksi}$$

Note that σ_x & σ_y are the principal stresses, so $\theta_p = 0, 90°$.

Hence $\theta_s = 45°, 135°$ or $\pm 45°$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 1.5 \text{ Ksi}$$

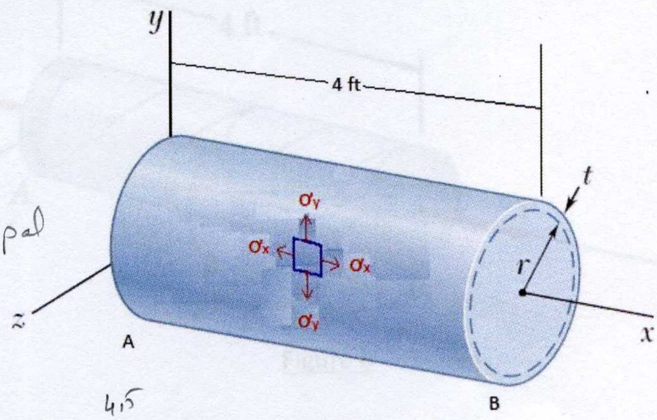
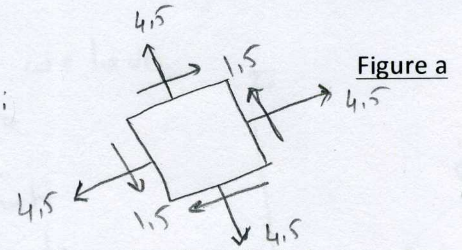


Figure a



b- When the vessel is now simultaneously pressurized as described above and compressed by an external axial force of 10 kip at each end, calculate σ_x and σ_y **(8 pts)**

Axial load: $\sigma_F = -\frac{F}{A}$ (-ve since it is compressive)

$F = 10$ Kip

$A = \pi(r_o^2 - r_i^2)$ where $r_o = r + t$ & $r_i = r$

$= \pi(5.25^2 - 5^2) = 8.05 \text{ in}^2$

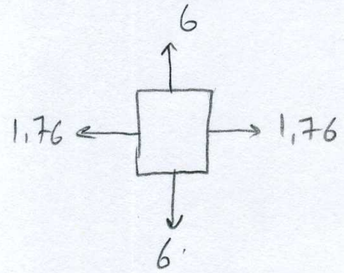
$$\sigma_F = \frac{-10}{8.05} = -1.242 \text{ Ksi}$$

Note that σ_F acts along x-axis, so now:

$\sigma_x = 3 - 1.242 = 1.76$ Ksi

$\sigma_y = 6$ Ksi (unchanged)

c- Consider now that the pressure vessel is spirally welded at an angle β as shown in Figure b and that the loading is such that σ_x =1.76 ksi, σ_y =6 ksi and τ_{xy}=0. Assuming that the tensile normal stress perpendicular to the weld cannot exceed 3.15 ksi, perform a *Mohr's circle* graphical analysis to estimate the maximum allowable value of β. **(12 pts)**



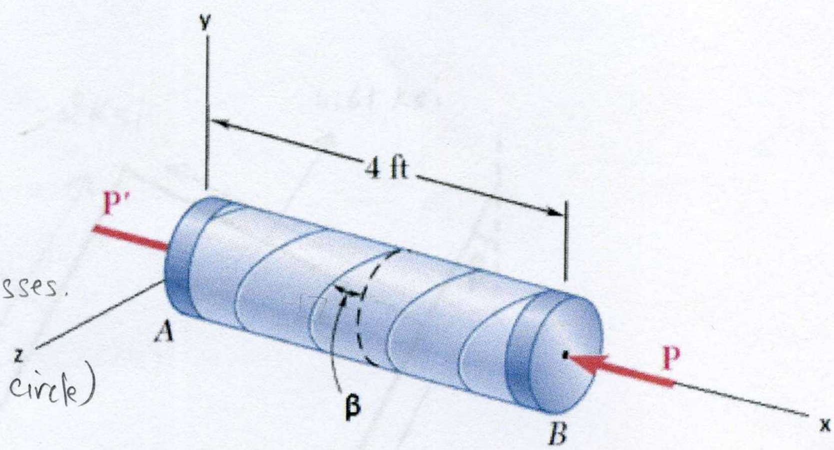


Figure b

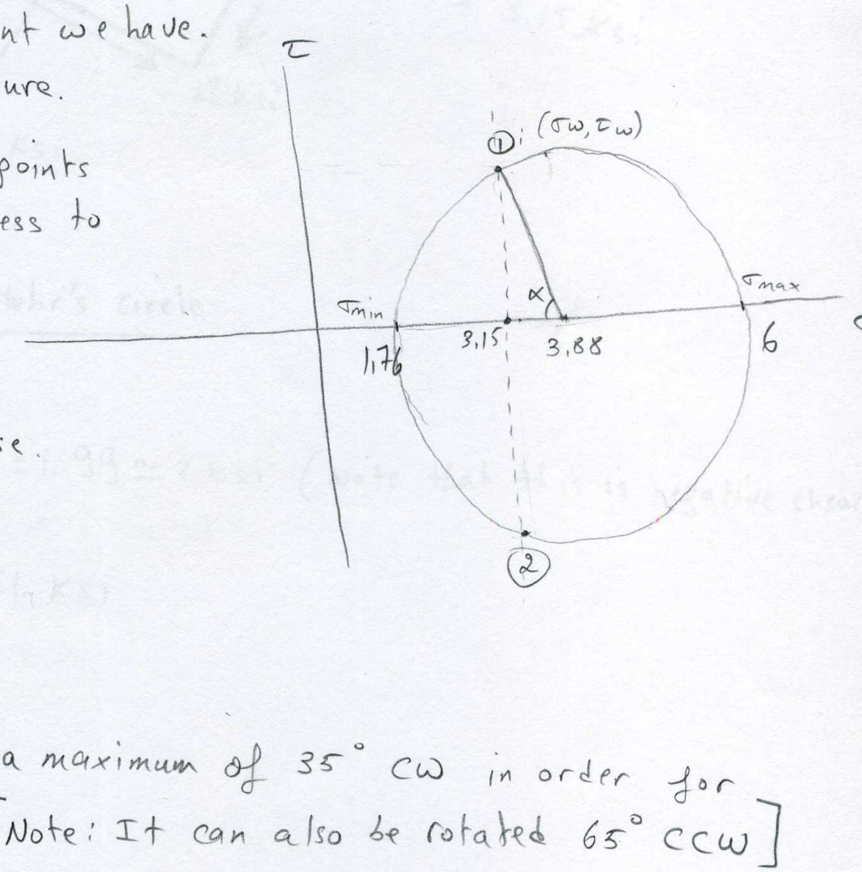
σ_x & σ_y are the principal stresses.

$\sigma_{av} = \frac{\sigma_x+\sigma_y}{2} = 3,88$ ksi (center of circle)

$R = \frac{\sigma_x-\sigma_y}{2} = 2,121$ ksi

we draw Mohr's circle using the element we have.
we add the line $\sigma = 3,15$ ksi on figure.
This line intersects the circle on 2 points ① & ② & represents max normal stress to the weld.

We are interested in point ① since we will rotate the element clockwise



$\cos\alpha = \frac{(3,88-3,15)}{2,12} \Rightarrow \alpha = 70^\circ$

$\alpha = 2\beta \Rightarrow \beta = 35^\circ$

Hence, the element can be rotated a maximum of 35° CW in order for $\sigma_{(weld)}$ not to exceed 3,15 (ksi). [Note: It can also be rotated 65° CCW]

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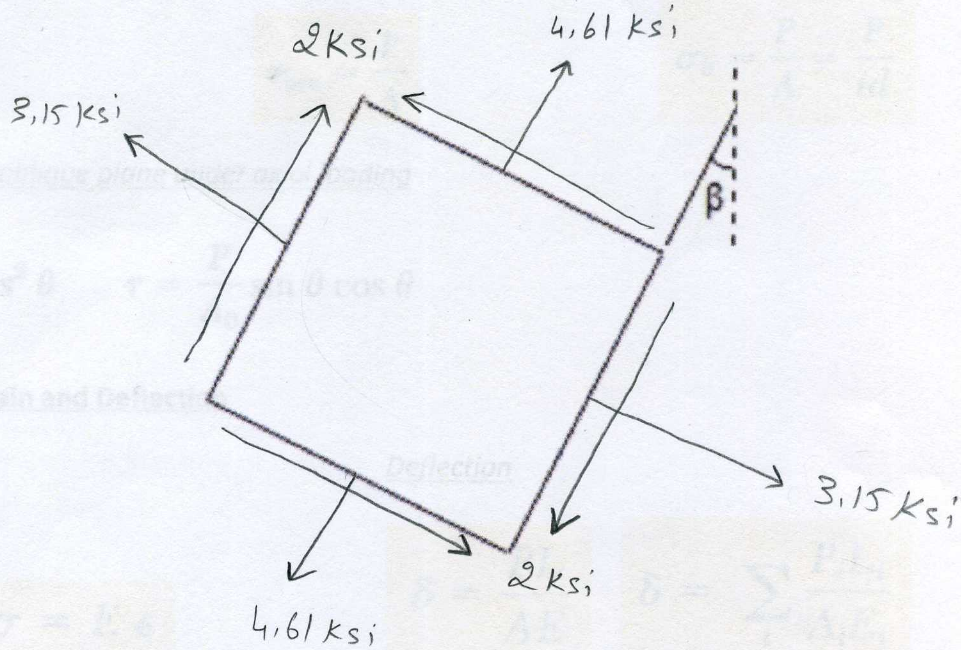
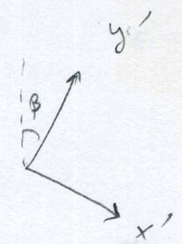
d- For the element shown here at the maximum allowable value of β, draw in all the stress components, paying attention to sense (tension, compression, direction of shear). **(8 pts)**





From Mohr's circle

$\sigma_{x'} = 3,15$ ksi

$|\tau_{x'y'}| = R\sin(\alpha) = 2,12\sin(70) = 1,99 \simeq 2$ ksi (Note that this is negative shear)

$\sigma_{y'} = 3.88 + R\cos(70) = 4.61$ ksi