Solution of Midterm 1

ME 106, Fall 2015

Problem 1: ***(Total 5 points)*** This will not work. Suppose we have a perfect pump producing a perfect vacuum (absolute zero pressure). The pressure at the water level is atmospheric since the well is open. Using the hydrostatic relation between the pump (0) and water level (1)

$$P_0 = P_1 - \gamma h_{max}, \quad P_0 = 0, \quad \Rightarrow \quad h_{max} = \frac{P_0}{\gamma} = \frac{101000(\text{pa})}{998(\frac{\text{kg}}{\text{m}^3}) \times 9.81(\frac{\text{m}}{\text{s}^2})} = 10.31(\text{m}).$$

Thus a perfect pump will draw the water up only 10.31m, not the 18m required.

Problem 2: ***(Total 9 points)*** Bernoulli's equation represents conservation of energy or conservation of total pressure *(1 point)*. The P is static or thermodynamics pressure *(1 point)*, $\frac{1}{2}\rho V^2$ is dynamic pressure or specific kinetic energy *(1 point)*, and $\rho g z$ is the hydrostatic pressure or specific potential energy due to gravity *(1 point)*. The assumptions are

- Steady state or $\partial/\partial t = 0$. *(1 point)*
- Incompressible, or ρ is constant. *(1 point)*
- Inviscid, or $\mu = 0$. *(1 point)*
- No source of heat or isothermal (also no external work done to the flow). *(1 point)*
- Relation holds along a streamline or in 1D flow. *(1 point)*

Problem 3: ***(Total 12 points)*** Use Bernoulli's equation along a streamline from the top surface (1) to the centerline of the exit jet(2).

$$P_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2, \quad \textit{(3 point)}$$
$$P_1 = P_{atm}, z_1 - z_2 = H, V_1 = 0 \quad \textit{(3 point)}$$

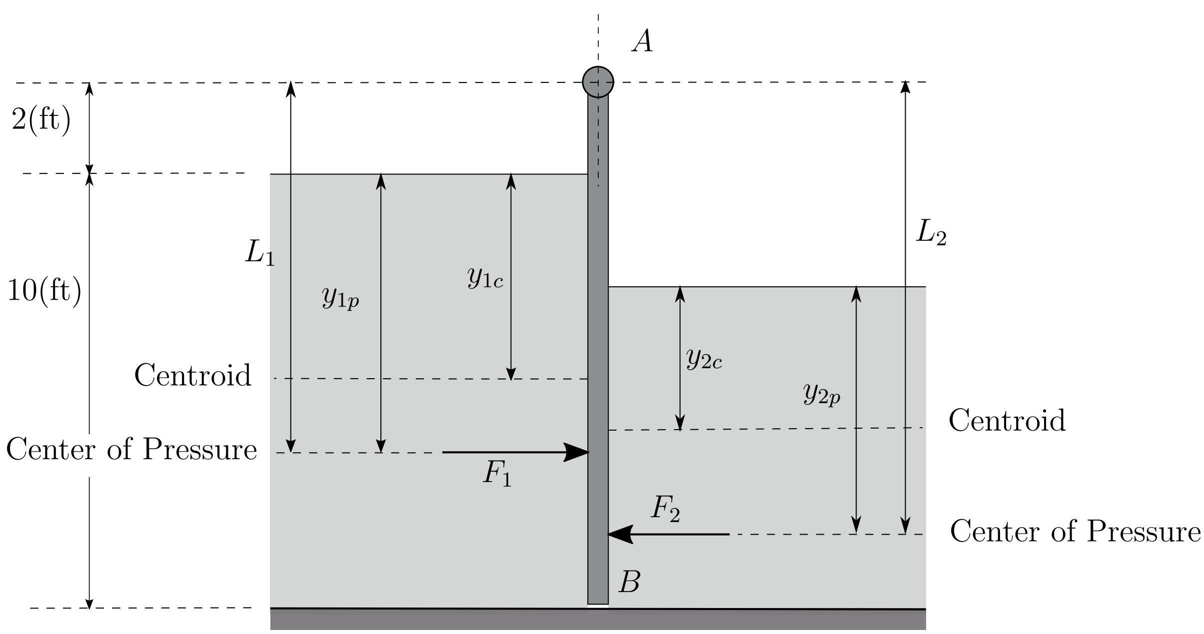
Surface tension around the jet increases the pressure inside the jet (P_2) according to

$$P_2 - P_{atm} = \frac{2\sigma}{D} \quad \textit{(3 point)}$$

substituting the above into Bernoulli's equation and solving for V_2 we obtain

$$V_2 = \sqrt{\frac{1}{\rho}\left(\frac{-4\sigma}{D} + 2\gamma H\right)} \quad \textit{(3 point)}$$

Problem 4: ***(Total 20 points)*** Let $b = 5$(ft) be the width of the gate into the page. The



resultant pressure on the left side is the pressure at the centroid $h_{1c} = 5$(ft)

$$P_{1c} = \gamma_w h_{1c} = \gamma(\text{lb/ft}^3) \times 5(\text{ft}) = 5\gamma(\text{lb/ft}^2), \quad \textit{(2 point)}$$

The resultant force F_1 is pressure P_{1c} times wet area A_1

$$A_1 = h_1 b = 10(\text{ft}) \times 5(\text{ft}) = 10b(\text{ft}^2), \quad \textit{(1 point)}$$

$$F_1 = P_{1c}A_1 = 5\gamma(\text{lb/ft}^2) \times 10b(\text{ft}^2) = 50\gamma b(\text{lb}). \quad \textit{(2 point)}$$

The resultant force F_1 acts at the centroid of the pressure prism (2/3 depth = 20/3 ft). This can also be derived as

$$y_{1p} = y_{1c} + \frac{I_{xx}^c}{y_{1c}A_1} = \frac{h_1}{2} + \frac{\frac{1}{12}h_1^3 b}{\frac{h_1}{2}(h_1 b)} = \frac{2}{3}h_1 = \frac{2}{3} \times 10 = 6.67(\text{ft}). \quad \textit{(3 point)}$$

Hence the moment arm about the hinge is $L_1 = 2(\text{ft}) + 6.67(\text{ft}) = 8.67(\text{ft})$ *(1 point)*.

On the right side, the centroid is at $h_{2c} = h/2$, and the pressure at centroid is P_{2c}, the wet area A_2 and the force from the right side F_2 are

$$P_{2c} = (SG)\gamma(h/2) = 1.025 \times \gamma(\text{lb/ft}^3)h/2 = 0.5125\gamma h(\text{lb/ft}^2), \quad \textit{(2 point)}$$
$$A_2 = hb(\text{ft}^2), \quad \textit{(1 point)}$$
$$F_2 = P_{2c}A_2 = 0.5125\gamma h(\text{lb/ft}^2)hb(\text{ft}^2) = 0.5125\gamma bh^2(\text{lb}). \quad \textit{(2 point)}$$

The resultant force F_2 acts at 2/3 depth (2h/3 ft), which can also be derived as

$$y_{2p} = y_{2c} + \frac{I_{xx}^c}{y_{2c}A_2} = \frac{h}{2} + \frac{\frac{1}{12}h^3 b}{\frac{h}{2}(hb)} = \frac{2}{3}h(\text{ft}). \quad \textit{(2 point)}$$

Hence the moment arm from point A is $L_2 = 12(\text{ft}) - h/3(\text{ft})$ *(1 point)*.

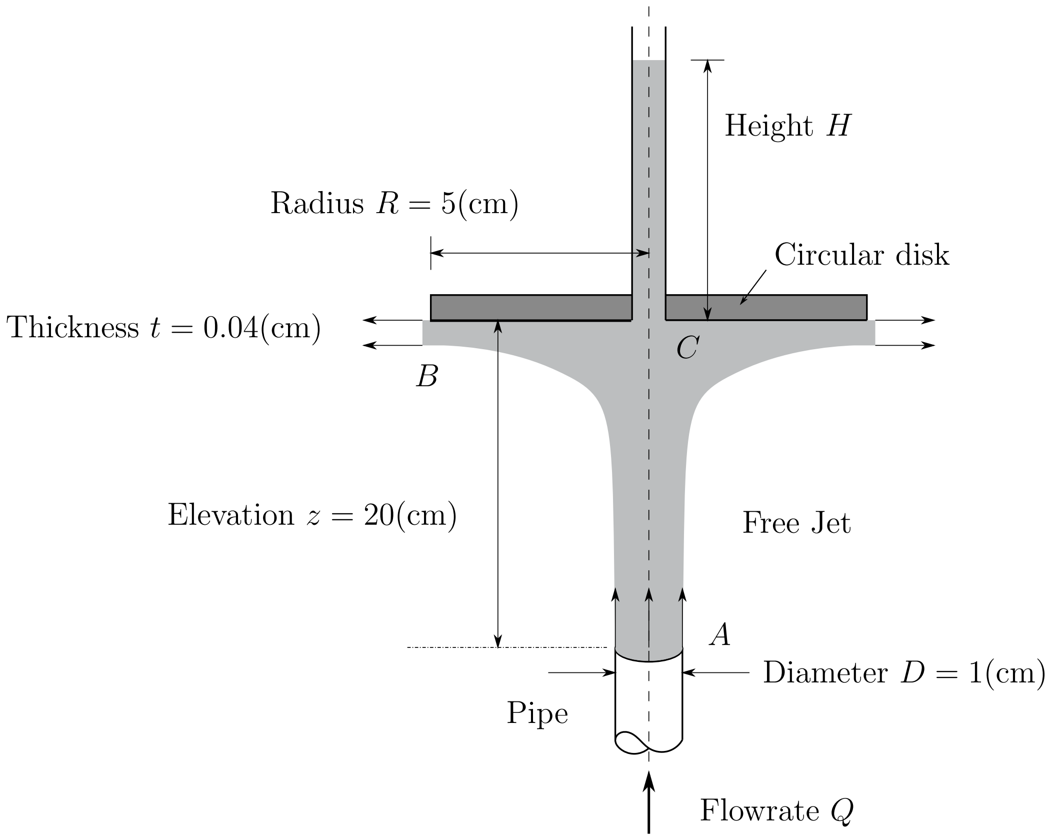
The balance of moments around point A is

$$\sum M_A = 0, \quad \Rightarrow \quad F_1 L_1 - F_2 L_2 = 0,$$
$$50\gamma b(\text{lb}) \times 8.67(\text{ft}) = 0.5125\gamma bh^2(\text{lb}) \times (12 - h/3)(\text{ft}) \quad \textit{(2 point)}$$

Simplifying and solving $h^2(36 - h) = 2537.56$ yields $h = 9.85(\text{ft})$ *(1 point)*.

Problem 5: ***(Total 20 points)*** **(a)** From conservation of mass $Q = V_A A_A = V_B A_B$ *(2 point)*. The area of the pipe outlet is $A_A = \frac{\pi}{4}D^2 = 7.8538 \times 10^{-5}(\text{m}^2)$ *(1 point)* and the area of horizontal output at the edge of circular disk is $A_B = 2\pi Rt = 1.2566 \times 10^{-4}(\text{m}^2)$ *(1 point)*. Writing Bernoulli between point A and B *(1 point)*

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B.$$



Since both points A and B are free jets, $P_A = P_B = P_{atm}$ *(1 point)* and the cancel from the equation. Also set $z_A = 0$ and $z_B = z$ *(1 point)*, the elevation of point B with respect to the pipe outlet. Moreover, from conservation of mass

$$V_A = \frac{Q}{A_A}, \quad V_B = \frac{Q}{A_B}. \quad \textit{(2 point)}$$

Substituting velocities in Bernoulli yields

$$\frac{Q^2}{2gA_A^2} = \frac{Q^2}{2gA_B^2} + z. \quad \textit{(1 point)}$$

Solving for Q gives *(2 point)*

$$Q = \sqrt{\frac{2gz}{A_A^{-2} - A_B^{-2}}} = \sqrt{\frac{2 \times 9.81(\mathrm{m/s^2}) \times 0.2(\mathrm{m})}{(7.8538 \times 10^{-5}(\mathrm{m^2}))^{-2} - (1.2566 \times 10^{-4}(\mathrm{m^2}))^{-2}}} = 2 \times 10^{-4}(\mathrm{m^3/s}).$$

(b) Writing Bernoulli equation between point (A) and (C)

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_C}{\gamma} + \frac{V_C^2}{2g} + z_C. \quad \textit{(1 point)}$$

The flow at (A) is free jet so the gage pressure $P_A = P_{atm} = 0$ *(1 point)*. However, point (C) is stagnation point, hence P_C is the stagnation pressure, and $V_C = 0$ *(1 point)*. The height of manometer is $H = P_C/\gamma$ *(2 point)*, so

$$H = \frac{P_C}{\gamma} = \frac{V_A^2}{2g} - z. \quad \textit{(1 point)}$$

Also $V_A = Q/A_A = 2 \times 10^{-4}(\mathrm{m}^3/\mathrm{s})/7.8538 \times 10^{-5}(\mathrm{m}^2) = 2.545(\mathrm{m/s})$ *(1 point)*. Thus

$$H = \frac{(2.545(\mathrm{m/s}))^2}{2 \times 9.81(\mathrm{m/s}^2)} - 0.2(\mathrm{m}) = 0.13(\mathrm{m}). \quad \textit{(1 point)}$$