# Hallatschek Spring 2015 Midterm 1

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## Problem 1

I will use the coordinate system below where $\hat{x}$ sits along the ramp and $\hat{y}$ points out of the ramp. As well, I set the (x, y) origin as the initial position of the cart.

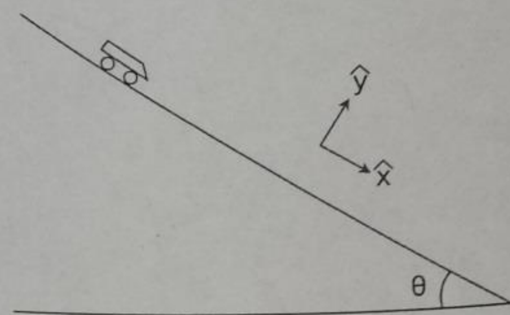



Figure 1: Coordinate Definitions

### Part a

The accelerations of the cart and the ball in the x direction are the same. The ball and the cart also share the same x velocity and x position when the ball is launched. Therefore, the ball will land back on the cart.

### Part b

We need to find the y position of the ball in the air as a function of time. The ball lands at time $t_1$, returning to $y = 0$.

$$a_y = -g\cos\theta$$

$$y(t) = v_\perp(t - t_0) + \frac{1}{2}a_y(t - t_0)^2$$

$$y(t_1) = 0 \Rightarrow v_\perp(t_1 - t_0) = -\frac{1}{2}a_y(t_1 - t_0)^2$$

$$t_1 - t_0 = \frac{-2v_\perp}{a_y} = \frac{2v_\perp}{g\cos\theta}$$

$$t_1 = t_0 + \frac{2v_\perp}{g\cos\theta}$$

### Part c

To find the distance the cart moves, we can use the kinematics equations in the x direction.

$$a_x = g\sin\theta$$

$$\Delta x \equiv x(t_1) - x(t_0) = \frac{1}{2}a_x t_1^2 - \frac{1}{2}a_x t_o^2 = \frac{1}{2}a_x(t_1^2 - t_0^2)$$

$$\Delta x = \frac{1}{2}a_x(t_1 - t_0)(t_1 + t_0)$$

$$\Delta x = \frac{1}{2}g\sin\theta\frac{2v_\perp}{g\cos\theta}\left(2t_0 + \frac{2v_\perp}{g\cos\theta}\right)$$

$$\Delta x = 2v_\perp\tan\theta\left(t_0 + \frac{v_\perp}{g\cos\theta}\right)$$

## Problem 2

### Part a

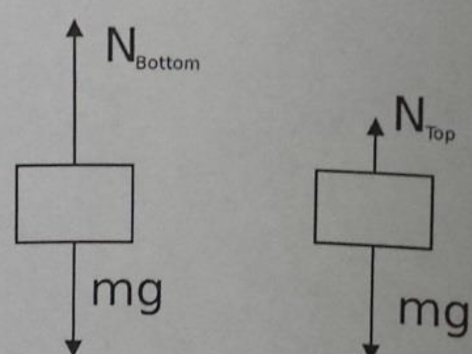



Figure 2: Free Body Diagrams. To the left is the FBD at the bottom of the wheel. The right is the FBD at the top of the wheel.

The acceleration at the bottom of the Ferris wheel is $v^2/R$ upwards. The acceleration the top of the Ferris wheel is $v^2/R$ downwards. The above FBDs describe the forces on e person at the top and the bottom of the wheel. The following force equations relate e forces and acceleration using the upwards direction as positive:

$$\sum F_{bottom} = N_b - mg = ma_b = m\frac{v^2}{R}$$

$$\sum F_{top} = N_t - mg = ma_t = -m\frac{v^2}{R}$$

The normal force is the force read by the scale. At the bottom, $N_b = 1.5mg$

$$N_t = -m\frac{v^2}{R} + mg = -(N_b - mg) + mg = (-1.5 + 1 + 1)mg = .5mg$$

### Part b

Use the force equation when the person is at the bottom of the wheel.

$$\sum F_{bottom} = N_b - mg = .5mg = m\frac{v^2}{R} \Rightarrow R = \frac{2v^2}{g}$$

### Part c

The condition for weightlessness is that gravity provides all of the force acting on a person. Thus, the normal force is zero. This would happened at the top of the curve when the acceleration need to go in a circle is the same as the acceleration due to gravity.

$$\frac{v_{max}^2}{R} = g \Rightarrow v_{max} = \sqrt{gR}$$

## Problem 3

For this problem, I will define $\hat{x}$ to the right and $\hat{y}$ up. The below free body diagram describes the forces acting on the crate. Notice that the only force in the x direction is the force of static friction. The static friction does its best to match the crate's acceleration to the truck's acceleration. (This is so the crate doesn't slide on the truck.) The maximum magnitude of static friction is $\mu_s N$.

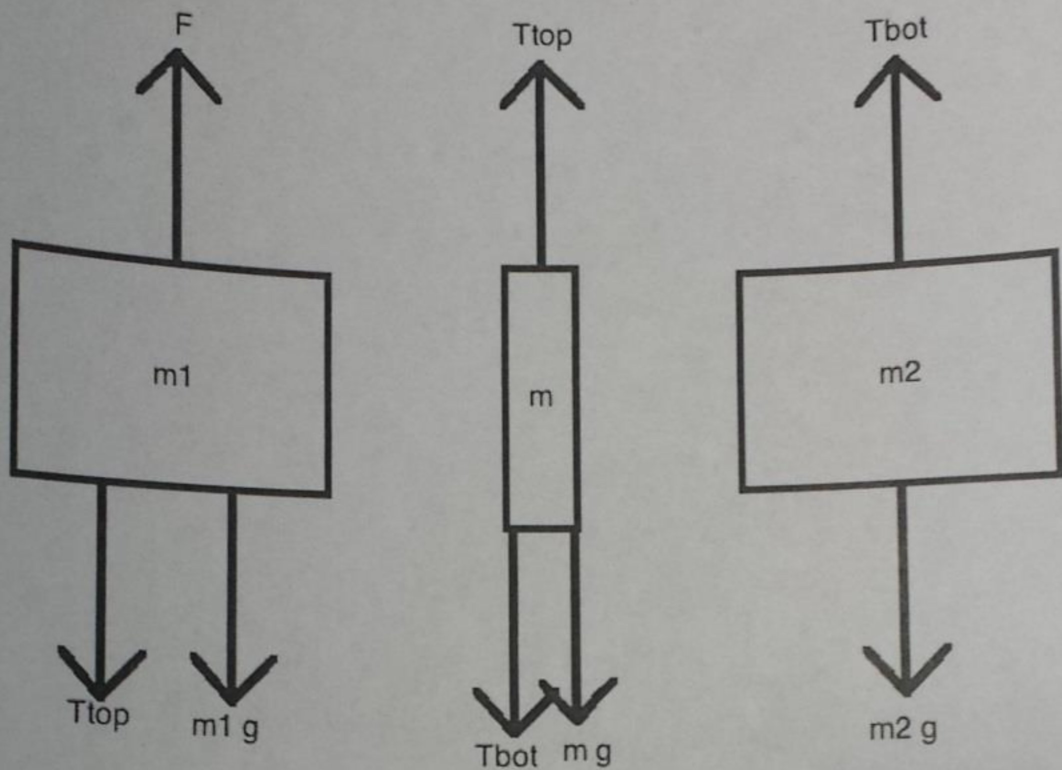
Summing the forces in the x and y directions results in the following equations:

$$\sum F_x = -f_{static} = ma_x$$

$$\sum F_y = N - mg = ma_y = 0$$

## Problem 4

### Part (a):




### Part (b):

$$\sum F_{ext} = M_{tot}a$$

$$F - (m_1 + m + m_2)g = (m_1 + m + m_2)a$$
$$a = (F/(m_1 + m + m_2)) - g$$

### Part (c):

Applying Newton's second law to the entire rope gives us:

$$T_{top} - T_{bot} - mg = ma$$

To eliminate the unknown $T_{bot}$, we write Newton's law for $m_2$:

$$T_{bot} - m_2 g = m_2 a$$

$$T_{bot} = m_2(g + a)$$

Plugging that into the first equation gives:

$$T_{top} = m_2(g + a) + m(g + a) = (m_2 + m)(g + a)$$

$$T_{top} = (m_2 + m)(F/(m_1 + m + m_2))$$

## Part (d):

Our calculation would be identical to that in part (c), except $T_{top} \to T_{mid}$ and $m \to m/2$. Therefore our final answer is

$$T_{mid} = (m_2 + m/2)(F/(m_1 + m/2 + m_2))$$

## Part (e):

Tension is greatest at the top of the rope, so we simply plug $F_c$ in for $T_{top}$ and solve for $m_2$.

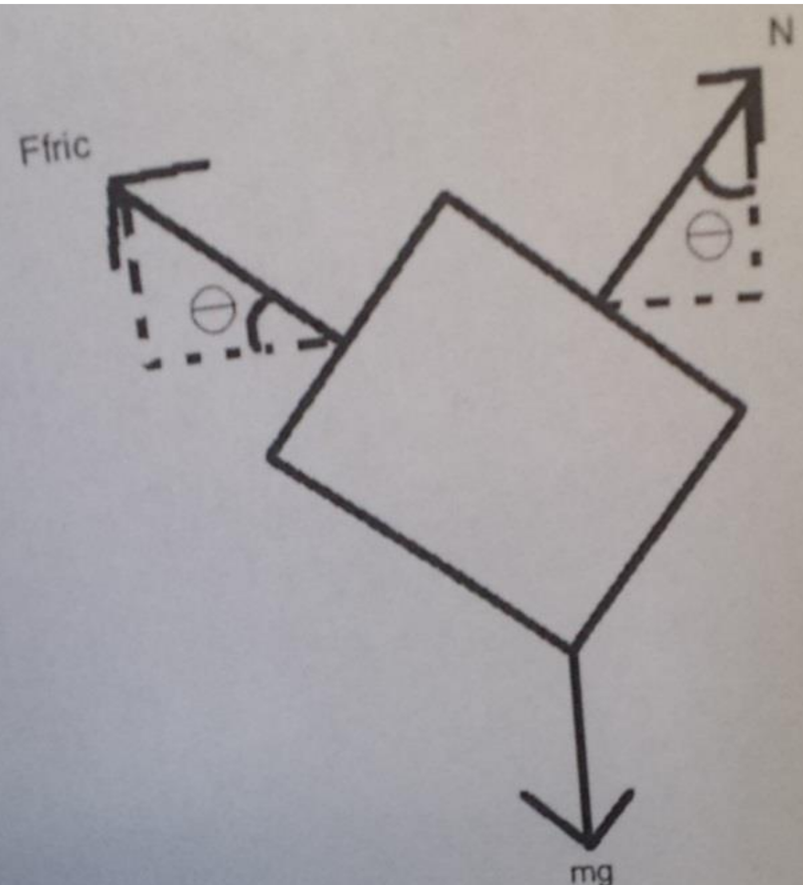
$$F_c = (m_2 + m)(F/(m_1 + m + m_2))$$

$$m_2 = (m(F - F_c) - m_1 F_c)/(F_c - F)$$

# Problem 5

We choose our axes such that positive x is in the direction of F and gravity is pulling in the negative y direction. This will make it easier to describe the acceleration, which makes solving Newton's equations easier.

Let's first solve for the minimum force required to have the block not slip. Then friction is pointing up and to the left on the block.




Newton's second law for the block, in the x and y direction, are

$$\sum F_x = N\sin\theta - \mu_s N\cos\theta = ma$$

$$\sum F_y = N\cos\theta + \mu_s N\sin\theta - mg = 0$$

Note that the x acceleration must match that of the large block, a. Likewise, the y acceleration must be 0 because the large block is not accelerating in the y direction, so the small block cannot either. We solve this system of equations for $a_{min}$:

$$a_{min} = g(\sin\theta - \mu_s\cos\theta)/(\cos\theta + \mu_s\sin\theta)$$

We find $F_{min}$ by noting that

$$F_{min} = (m+M)a_{min} = (m+M)g(\sin\theta - \mu_s\cos\theta)/(\cos\theta + \mu s\sin\theta)$$

Finally, we quickly find the maximum force such that the block does not slide up, $F_{max}$, by noting the one way that our procedure above would have changed. All we would have done differently is point friction down and to the right. That is, we would point friction in exactly the opposite direction. This would have just switched the sign of our friction terms in our Newton's second law equations. Therefore $F_{max}$ will be the same as $F_{min}$ if we switch the sign of all the $\mu_s$.

$$F_{max} = (m + M)g(\sin\theta + \mu_s \cos\theta)/(\cos\theta - \mu_s \sin\theta)$$