EECS 126 Probability and Random Processes University of California, Berkeley: Spring 2015
Abhay Parekh February 17, 2015

Midterm Exam

Last name	First name	SID

Rules.

- You have 80 mins (5:10pm - 6:30pm) to complete this exam.
- The exam is not open book, but you are allowed half a sheet of handwritten notes; calculators will be allowed.
- No form of collaboration between the students is allowed. If you are caught cheating, you may fail the course and face disciplinary consequences.

Please read the following remarks carefully.

- Show all work to get any partial credit.
- Take into account the points that may be earned for each problem when splitting your time between the problems.

Problem	Points earned	out of
Problem 1		40
Problem 2		25
Problem 3		25
Problem 4		10
Extra Credit		10
Total		100+10

Problem 1 [40] Answer the following problems briefly but clearly.

(a) [10] Let X be a continuous random non-negative variable (i.e. it has a density function, $f_X(x)$) with strictly increasing cdf F_X. Find the density functions of $Y = \sqrt{X}$ and $Z = F_X(X)$.

(b) [10] X, Y, Z are iid uniform r.v. over $[0, 1]$.

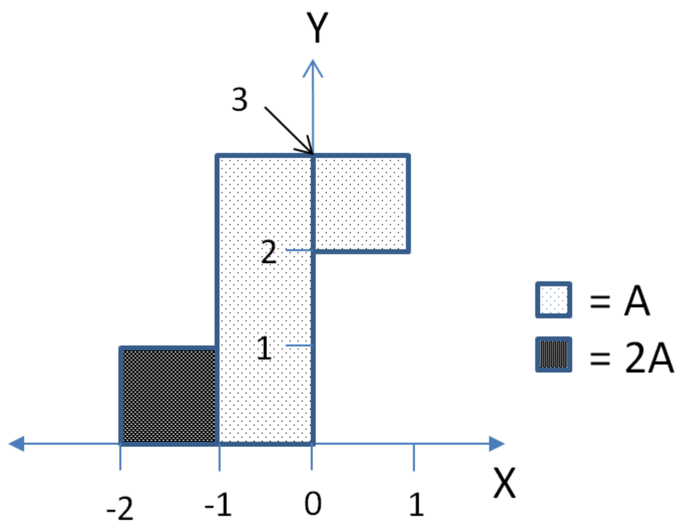
1. Determine the density function for $U = X + Y + Z$
2. Let $V = XY$ and find the joint pdf $f_{VZ}(v, z)$.

(c) [10] An expensive blood test must be performed on n individuals. The probability that any individual is infected is p. Each person can be tested separately but to save money, the individuals are grouped groups of k (assume that $\frac{n}{k}$ is an integer), and the test applied to each group. If the test is negative, all the individuals in the pool are negative. If it tests positive at least one of the individuals is positive and k more tests have to be performed to determine the status of each of the members of the group.

Find the expected number of tests necessary to test the n individuals under this strategy.

(d) [10] Let X and Y be random variables and define $U = X + Y$ and $V = X - Y$.

1. Find $cov(U, V)$.
2. If X, Y are exponentially distributed with parameters λ_x and λ_y respectively, can U and V be independent?



Problem 2 [25] X and Y are two random variables with joint distribution $f_{XY}(x,y)$ as shown in the figure above. Find the value of A, $cov(X,Y)$ and determine $f_{X|Y}(x|y)$ for all values of y.

Problem 3: Coin Sequences[25]

(a) [6] Bob flips a fair coin coin. Let X be the number of flips until he gets the sequence HH (gets two consecutive heads). Let Y be the number of flips until he gets the sequence TH. Find $E[X]$ and $E[Y]$.

(b) [6] Bob plays the following game with Alice who is watching him flip a fair coin. Bob wins if the sequence HT comes before HH, and Alice wins otherwise. What is the probability that Bob will win this game?

(c) [6] Bob flips a coin which lands heads with probability p. What is the probability that he will observe r Heads before he observes s Tails?

(d) [7] The bias of Bob's coin (prob coin will land on heads) is uniformly distributed over $[0, 1]$. He tosses the coin n times. What is the expected value and variance of, X, the total number of heads?

Problem 4 [15]

A company has created a test to determine if a worker is proficient at a particular task. There are N workers, m of who are proficient. Workers are picked at random and tested, and X is the number of workers needed to be tested in order to find some fixed number, $k(\leq m)$, proficient workers. Assume that once a worker has been tested they are removed from the pool and cannot be selected again.

(a) [8] Find $p_X(x)$. Your answer for the first part should have no summations but can have terms such as $\binom{N+m}{k+r}$. Hint: Find the right quantity to condition on.

When $X = x$ the xth worker tested will be proficient. He is equally likely to be any of the m proficient workers so the probability that a particular one is picked is $\frac{m}{N}$.

(b) [7] Find $E[X]$. Hint: Use linearity of expectations.

Extra Credit: [10] No partial credit for this problem.

Acme Inc has the following interview policy for filling a single position. n candidates are interviewed one at a time. Each candidate receives a score (no ties and a higher score is better). A candidate is either accepted or rejected for the job immediately after their interview. The company decides to interview and reject the first m candidates. After the mth candidate, the policy is to hire the first candidate who scores higher than the all the previous candidates interviewed.

Let E be the event that the best candidate out of the n is hired, and let E_i be the event that the ith candidate is best candidate and is hired. Find $P(E_i)$ and $P(E)$. Note, $i > m$. Also, you can leave your answer as a sum over i or try to bound it.