Friday, October 7, 1:00–2:00 pm, 1994.

Please write all answers in the space provided. If you need additional space, write on the back sides. Indicate your answer as clearly as possible for each question. Write your name at the top of each page as indicated.

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1. (20 points) Dynamic Analysis and Gait

Figure 1 shows a free-body diagram of an idealization of the shank during gait at an angle q to the vertical. For simplicity, the foot has been ignored, and the ground reaction force R is assumed to act at the ankle. Also shown are the resultant force Fres and moment Mres acting at the knee joint. Express the resultant moment Mres in terms of the ground reaction force R, the mass of the shank m, the moment of inertia of the shank with respect to its center of gravity Icg, and the accelerations of the shank (acg and a). Note: all unknowns have been drawn in their assumed positive directions. Express vectors in X-Y components.

Figure 1: Free-body diagram and kinematic diagram for the shank during gait.

2. (20 points; 10 points each) Forces on Bones

(i) Typical loads that act on the femoral mid-diaphysis during gait result in tensile stresses acting on the lateral or medial aspect of the mid-diaphysis?

(ii) In reconstructing the hip joint, surgeons sometimes move the point of attachment of the abductors (i.e. the greater trochanter) laterally. Will this cause the magnitude of the joint contact force to increase or decrease?

3. (15 points; 5 points each) Mechanical Behavior of Bone

(i) On the same graph, plot stress-strain curves (with strains from 0-50%) for human trabecular bone in uniaxial compression for two different apparent densities; indicate which curve is for the higher apparent density.

(ii) Cortical bone, as tested in the laboratory (i.e. devitalized), has a higher yield strength in compression than in tension. What about trabecular bone? As part of your answer, draw a plot of yield strength vs. modulus for tension and compression.

(iii) On the same graph, sketch a tensile stress-strain curve for cortical bone, one for low strain rate loading and one for high strain rate loading; indicate which curve is for the higher strain rate.

4. (15 points) Multiaxial Failure Criteria

The Tsai–Wu multiaxial failure criterion in two dimensions (1–2 plane) can be written as follows:

F1s1 + F2s2 + 2F12s1s2 + F11s12 + F22s22 + F33s32 = 1,

where the F terms are the Tsai–Wu coefficients, and the s terms are the stresses (subscripts 1 and 2 refer to normal stresses in the 1 and 2 directions, respectively; subscript 3 refers to shear stresses in the 1–2 plane). Apply this criterion (i.e. write out the equation) to the uniaxial compression test on a piece of human cortical bone, loaded along its longitudinal material axis. Assume typical values of the appropriate failure stresses.

5. (30 points) Prosthesis Design

As a hip prosthesis designer, you are faced with the decision to add either a proximal–medial collar (Figure 2a) or proximal porous coating (Figure 2b). Your design rationale is to use whichever design has lower bending stresses in the stem at the cross–section x–x.

To solve this design problem, assume that there are only three forces acting on the prosthesis in vivo: (i) the joint contact force which has known magnitude H and acts at a known angle q as shown; (ii) a distal contact force D that is horizontal and acts at point d but has an unknown magnitude; and (iii) a proximal supporting force P, that acts through either point a (if a collar is used) or b (if porous coating is used). The magnitude and direction of the proximal support force are not known. Draw in the line of action for the force P in each diagram and determine which design should be used by completing the free–body diagram analysis. Hint: this solution does not require any equations...

Figure 2: Two proposed designs for a hip implant, one with a proximal–medial collar (left) and the other with proximal porous coating (right).