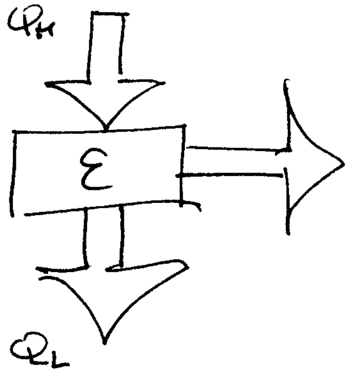
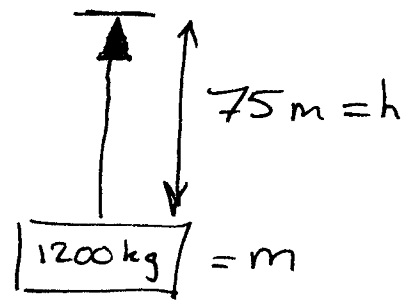
Peter Battaglino

1. $T_H = 400°C$



W is used to lift



$T_L = 0°C$ $\qquad\qquad \varepsilon = 0.65\, \varepsilon_{Carnot}$

$W = mgh, \qquad Q_L = m_{ice} L_f^{water}$

$Q_H = Q_L + W,$

$W = \varepsilon Q_H \longrightarrow Q_H = \frac{1}{\varepsilon} W$

$\frac{1}{\varepsilon} W = Q_L + W \longrightarrow Q_L = \left(\frac{1}{\varepsilon} - 1\right) W = \frac{1-\varepsilon}{\varepsilon} W$

$Q_L = m_{ice} L_f^{water} = \frac{1-\varepsilon}{\varepsilon} W = \frac{1-\varepsilon}{\varepsilon} mgh$

$m_{ice} = \frac{1-\varepsilon}{\varepsilon} \frac{mgh}{L_f^{water}}, \qquad \varepsilon_{Carnot} = 1 - \frac{T_L}{T_H}$

$= 1 - \frac{273K}{673K} = 0.594$

$\varepsilon = 0.65\, \varepsilon_{Carnot} = \boxed{0.386 = \varepsilon}$

$m_{ice} = \frac{1-0.386}{0.386} \frac{1200\,kg \cdot 9.81\,m/s^2 \cdot 75m}{3.3 \times 10^5\,J/kg}$

$\boxed{m_{ice} = 4.2\,kg}$

Packard Phys. 7B MT1
Spring 2008 Solutions

Michelle Yong

Problem 2

Use energy, not forces. Assume all potential energy of the block becomes heat energy, which raises the temperature of the block + causes volume expansion.

$$\text{P.E. block} = mgh = mgL\sin\alpha = Q = mc\Delta T$$

$$\rho_{final} = \frac{m}{V_{final}} \quad ; \quad V_{final} = V_0(1+\gamma\Delta T)$$

$$V_0 = l_0^3$$

$$\Delta T = \frac{gL\sin\alpha}{c} \quad \Rightarrow \quad V_{final} = l_0^3\left(1+\frac{\gamma g L\sin\alpha}{c}\right)$$

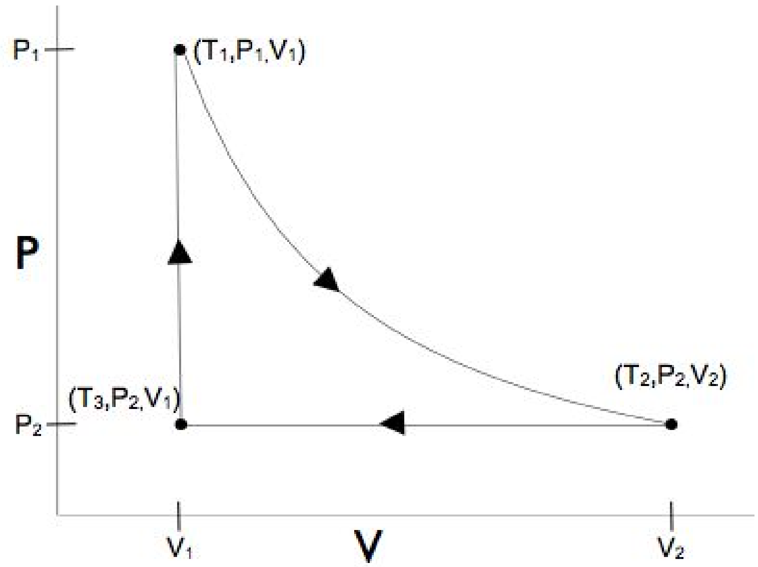
$$\rho_{final} = \underbrace{\frac{m}{l_0^3}}_{\rho_{initial}} \underbrace{\left(1+\frac{\gamma g L\sin\alpha}{c}\right)^{-1}}_{\# < 1}$$

$$\Rightarrow \quad \rho_{final} < \rho_{initial}$$

which is what we expect due to heating + volume expansion.

Problem 3

(a)



(b)

(i) For an adiabatic process we can immediately say:

$$Q_{1\to2} = 0 \tag{1}$$

To find $\Delta E_{1\to2}$ it's easiest to use $E = \frac{3}{2}Nk_bT$ for a monatomic ideal gas:

$$\begin{aligned}
\Delta E_{1\to2} &= \frac{3}{2}Nk_b\left(T_2 - T_1\right) && (2)\\
&= \frac{3}{2}\left(6.02\times10^{23}\right)\left(1.38\times10^{-23}\frac{J}{K}\right)\left(389K - 588K\right) && (3)\\
&= -2.48\times10^3\ J && (4)
\end{aligned}$$

As we expect, $\Delta E_{1\to2}$ is negative because the temperature decreases.

To find the work we'll use the first law:

$$\begin{aligned}
\Delta E_{1\to2} &= Q_{1\to2} - W_{1\to2} && (5)\\
\Rightarrow W_{1\to2} &= -\Delta E_{1\to2} && (6)\\
&= 2.48\times10^3\ J && (7)
\end{aligned}$$

As a check, we see the gas is expanding from $1 \rightarrow 2$ so $W_{1\rightarrow 2}$ should be positive.

(ii) We can directly calculate the work first:

$$
\begin{aligned}
W_{2\rightarrow 3} &= \int_{V_2}^{V_3} P dV && (8)\\
&= P_2 \left(V_3 - V_2\right) && (9)
\end{aligned}
$$

but we don't yet know P_2 or any of the volumes.

V_3 is easily found from the ideal gas law:

$$
\begin{aligned}
V_3 = V_1 &= \frac{N k_b T_1}{P_1} && (10)\\
&= \frac{\left(6.02 \times 10^{23}\right)\left(1.38 \times 10^{-23}\frac{J}{K}\right)\left(588\ K\right)}{1.01 \times 10^5\ Pa} && (11)\\
&= \ 4.84 \times 10^{-2} m^3 && (12)
\end{aligned}
$$

For the adiabatic process we have:

$$
P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \qquad (13)
$$

We can rewrite this in terms of temperatures using the ideal gas law $P = \frac{N k_b T}{V}$:

$$
\begin{aligned}
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} && (14)\\
\Rightarrow V_2 &= \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} V_1 && (15)\\
&= \left(\frac{588\ K}{389\ K}\right)^{\frac{3}{2}} 4.84 \times 10^{-2} m^3 && (16)\\
&= 8.99 \times 10^{-2}\ m^3 && (17)
\end{aligned}
$$

We can find P_2 from the ideal gas law:

$$P_2 = \frac{Nk_bT_2}{V_2} \tag{18}$$
$$= \frac{\left(6.02\times10^{23}\right)\left(1.38\times10^{-23}\,\frac{J}{K}\right)(389\ K)}{\left(8.99\times10^{-2}\ m^3\right)} \tag{19}$$
$$= 3.59\times10^{4}\ Pa \tag{20}$$

So now we can find the work using (9) :

$$W_{2\to3} = P_2\left(V_3 - V_2\right) \tag{21}$$
$$= \left(3.59\times10^{4}\ Pa\right)\left(4.84\times10^{-2}m^3 - 8.99\times10^{-2}\ m^3\right) \tag{22}$$
$$= -1.49\times10^{3}\ J \tag{23}$$

As expected, this work is negative because the gas is being compressed.

Again, we can write $E_{2\to3}$ in terms of the temperatures,

$$\Delta E_{2\to3} = \frac{3}{2}Nk_b\left(T_3 - T_2\right) \tag{24}$$
$$\tag{25}$$

To find T_3 we use the ideal gas law (again!):

$$T_3 = \frac{P_3V_3}{Nk_b} \tag{26}$$
$$= \frac{\left(3.59\times10^{4}\ Pa\right)\left(4.84\times10^{-2}m^3\right)}{\left(6.02\times10^{23}\right)\left(1.38\times10^{-23}\,\frac{J}{K}\right)} \tag{27}$$
$$= 209\ K \tag{28}$$

This makes sense as we know T_3 has to be less than T_1 and T_2 based on the PV diagram.

$$\Delta E_{2\to3} = \frac{3}{2}\left(6.02\times10^{23}\right)\left(1.38\times10^{-23}\,\frac{J}{K}\right)(209\ K - 389\ K) \tag{29}$$
$$= -2.24\times10^{3}\ J \tag{30}$$

This should be negative as the temperature is decreasing from $2 \to 3$.

Finally we get the heat from the first law:

$$
\begin{aligned}
Q_{2\to 3} &= \Delta E_{2\to 3} + W_{2\to 3} && (31)\\
&= -2.24\times 10^3\ J - 1.49\times 10^3\ J && (32)\\
&= -3.73\times 10^3\ J && (33)
\end{aligned}
$$

so heat leaves the gas as it is compressed at constant pressure.

(iii) Now we know everything we need to do $3 \to 1$

$$
\begin{aligned}
\Delta E_{3\to 1} &= \frac{3}{2} N k_b \left(T_1 - T_3\right) && (34)\\
&= \frac{3}{2}\left(6.02\times 10^{23}\right)\left(1.38\times 10^{-23}\frac{J}{K}\right)(588 - 209\ K) && (35)\\
&= 4.72\times 10^3\ J && (36)
\end{aligned}
$$

$$
W_{3\to 1} = \int_{V_3}^{V_1} P dV = 0 \qquad (37)
$$

$$
(38)
$$

$$
\begin{aligned}
Q_{3\to 1} &= \Delta E_{3\to 1} + W_{3\to 1} && (39)\\
&= 4.72\times 10^3\ J && (40)
\end{aligned}
$$

Because E is a state variable we better check:

$$
\Delta E_{1\to 2} + \Delta E_{2\to 3} + \Delta E_{3\to 1} = -2.48\times 10^3\ J - 2.24\times 10^3\ J + 4.72\times 10^3\ J = 0 \qquad (41)
$$

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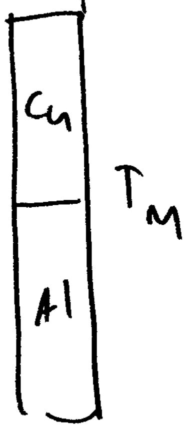
4) $T_H = 293\ K$



$T_L = 77\,K$

$$\frac{dQ}{dt} = \frac{k_{Cu} A}{l}(T_H - T_M) = \frac{k_{Al} A}{l}(T_M - T_L)$$

$$T_M = \frac{\cancel{\frac{A}{l}}\left(k_{Cu} T_H + k_{Al} T_L\right)}{\cancel{\frac{A}{l}}\left(k_{Cu} + k_{Al}\right)} = 213\ K$$

$$\frac{dQ}{dt} = 3.21\ \frac{J}{s}$$

$$Q = mL$$

$$\frac{dQ}{dt} = \frac{dn}{dt} L = \frac{dm}{dt} \cdot \frac{1}{M_w} L = \frac{dV}{dt}\rho \cdot \frac{1}{M_w} \cdot L$$

$$\frac{dV}{dt} = \frac{dQ}{dt}\frac{M_w}{\rho L} = 2.01 \times 10^{-8}\ \frac{m^3}{s} = 2.01 \times 10^{-5}\ \frac{L}{s}$$

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5. If $\ell \geq \lambda$, then the sound won't propogate.

$$\ell = \frac{1}{4\pi\sqrt{2}\, r^2 (N/V)}$$

$$r_{air} \approx 10^{-10}\ \text{m}$$

$$\rho = \frac{\text{mass of N air molecules}}{\text{volume they occupy}} = \frac{m_{air} \cdot N}{V}$$

So $\left(\frac{N}{V}\right) = \frac{1}{m_{air}} \rho$

$$\ell = \frac{1}{4\pi\sqrt{2}\, r_{air}^2 (\rho / m_{air})}$$

m_{air} in kilograms

$$\rho = \rho_0 e^{-mgy/kT}$$

ρ_0 ... density at sea level: take this to be at STP

$$\rho_0 = \boxed{\frac{N}{V}} m_{air} = \frac{P\, m_{air}}{kT}$$

$$PV = NkT$$

$$\frac{N}{V} = \frac{P}{kT}$$

where $P = 101325$ Pa

$T = 0°C = 273$ K

cutoff value

$$\ell = \frac{m_{air}}{4\pi\sqrt{2}\, r_{air}^2\, \rho_0} e^{+m_{air} g y / kT} = \lambda$$

$$e^{m_{air} g y / kT} = \frac{4\pi\sqrt{2}\, r_{air}^2 \rho_0 \lambda}{m_{air}}$$

$$\boxed{y_{max} = \frac{kT}{m_{air} g} \ln\left(\frac{4\pi\sqrt{2}\, r_{air}^2 \rho_0 \lambda}{m_{air}}\right)}$$

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5 continued:

$$y_{max} = \frac{kT}{M_{air}g} \ln\left(\frac{4\pi\sqrt{2}\, r_{air}^2 \rho_0 \lambda}{M_{air}}\right)$$

$k = 1.38 \times 10^{-23}$ J/K

$T = 273$ K

$M_{air} = 29u \times 1.66 \times 10^{-27} \frac{kg}{u} = 4.81 \times 10^{-26}$ kg

$g = 9.81$ m/s^2

$r_{air} \approx 10^{-10}$ m

$$\rho_0 = \frac{101325\ Pa \times 4.81 \times 10^{-26}\ kg}{1.38 \times 10^{-23} \frac{J}{K} \cdot 273\ K} = 1.29 \frac{kg}{m^3}$$

$\lambda = 0.2$ m

Putting it all together...

$$y_{max} = \frac{1.38 \times 10^{-23} \frac{J}{K} \times 273\ K}{4.81 \times 10^{-26}\ kg \times 9.81 \frac{m}{s^2}} \ln\left(\frac{4\pi\sqrt{2}\,(10^{-10} m)^2 \times 1.29 \frac{kg}{m^3} \times 0.2 m}{4.81 \times 10^{-26}\ kg}\right)$$

$$\boxed{y_{max} = 110\ km}$$