Physics H7C Midterm 1 Solutions

Problem 1.

a. We obviously cannot have moved the disk within the focal length, or the image would be virtual and we could not project it onto a screen. Therefore $o > f$ still. It is clear from a ray diagram that the image must get bigger ($d_2 > d_1$). The ray which passes through the middle of the lens gets more angled, so that ray travels further down before hitting the screen. Alternatively, with equations, the magnification is $m = -f/(o - f)$. Since o has gotten smaller, the denominator has gotten smaller. Therefore the magnification increases in magnitude, and again we get that $d_2 > d_1$.

Rubric (5 points):

- Two way to get full points. Way 1 (if ray diagrams are used):
  - 2 points total for one (or two) correct diagram(s) that clearly shows the desired property
  - 2 points for an explanation of what happens when the disk is moved
  - 1 point for the correct answer.
- Way 2 (if equations are used):
  - 2 points for the magnification equation
  - 2 points for the correct logic to get the answer
  - 1 point for the correct answer.
  - A maximum of 2/5 points total are awarded if the wrong answer arises because of confusion about the effect of the minus sign on the focal length.

b. The screen needs to be moved further away from the lens. Again, a ray diagram would suffice. The ray which passes through the middle of the lens gets more angled, so it takes more distance to intersect with a ray from the same point that entered the lens parallel to it. In equations, $1/o + 1/i = 1/f$. Since o has gone down, therefore i must go up to keep $1/f$ the same.

Rubric (5 points):

- Two ways to get full points. Way 1 (ray diagrams are used):
  - 2 points total for one (or two) correct diagram(s) (it is okay to refer to a diagram from part (a))
  - 2 points for an explanation of what happens when the disk is moved
  - 1 point for the correct answer.
- Way 2 (equations are used):
  - 2 points for the lens equation
  - 2 points for the right logic to get the answer
  - 1 point for the correct answer.

c. The magnification is given by $-f/(L_1 - f)$ when $o = L_1$. But the magnification is also given by $-d_1/D$ (negative because the image is inverted). Therefore

$$-\frac{f}{o-f} = -\frac{d_1}{D}\ . \tag{0.1}$$

Solving for f, we get

$$f = \frac{d_1 L_1}{d_1 + D} \tag{0.2}$$

Rubric (5 points):

- (2×) 2 points for each correct ratio between diameters and distances, up to 4 points total.
- 1 point for the correct answer.

Problem 2.

a. $P = (0.314)iV = 0.314$ W

Rubric (3 points):

- +2 points for $(0.314)iV$. (If you put $P = iV$ instead, only +1 point.)
- +1 point for correct numerical expression. (If 0.314 was omitted, this point is still awarded, as long as the numerical expression is consistent with $P = iV$.) No points if units are incorrect (but it is okay to leave it as Volts·Amps). Full points if the answer is rounded to one sigfig.

b. $I = P/A = 3.14 \times 10^3$ W/m^2

Rubric (3 points):

- +2 points for correct expression.
- +1 point for correct numerical evaluation.

c.

$$\frac{N}{\Delta t} = \frac{IA}{h\nu} = \frac{P}{h\nu} \tag{0.3}$$

Using $\nu = c/\lambda$, we get

$$\frac{N}{\Delta t} = \frac{P\lambda}{hc} \approx \frac{(3.14 \times 10^{-1})(5.50 \times 10^{-7})}{(6.28)(1.1 \times 10^{-34})(3 \times 10^{8})} \approx \frac{1 \times 5}{2 \times 3} \times 10^{18}\ \text{s}^{-1} \approx 8.3 \times 10^{17}\ \text{s}^{-1} \tag{0.4}$$

Rubric (4 points):

- +1 point for the right expression for $N/\Delta t$.
- +1 point for converting 550 nm to meters at any point in the calculation.
- +1 point for a reasonable attempt to numerically evaluate the result.
- +1 point for the correct answer, or something of the order of magnitude of 10^{18} per second.

d. This is the same as asking how many photons are emitted in the time $\Delta t = (1\text{ meter})/(3 \times 10^8\text{ m/s})$. The answer is

$$N = \frac{N}{\Delta t}\Delta t = (8.3 \times 10^{21}\ \text{s}^{-1})\frac{(1\text{ m})}{3 \times 10^8\text{ m/s}} \approx 3 \times 10^9 \tag{0.5}$$

Rubric (3 points):

- +2 points for a correct expression for N.
- +1 point for the right order of magnitude (if part (c) is incorrect, still award +1 point if the order of magnitude is 8 orders less than the incorrect answer from (c).)

e. The radiation pressure is $\mathscr{P} = 2I/c$. Then

$$\mathscr{P} = \frac{2(3.14 \times 10^3 \text{ W/m}^2)}{3 \times 10^8 \text{ m/s}} \approx 2 \times 10^{-5} \text{ W} \cdot \text{s/(m}^3) \quad (0.6)$$

(Note: since a Watt measures energy per second, and energy is force times distance, therefore 2×10^{-5} W · s/(m^3) = 2×10^{-5} N/(m^2), which are the right units for pressure.)

Rubric (5 points):

- 3 points for $\mathscr{P} = 2I/c$. (Only one point awarded out of three for $\mathscr{P} = I/c$.)
- 1 for an attempt at a numerical answer.
- 1 for an answer that is 8 orders of magnitude less than I.

f. 550 nm is green light.

Rubric (2 points):

- +2 points for an answer containing "green".
- If green is not the given solution, 1 point for yellow. (This partial credit is awarded also if the spectrum is written but the answer is wrong).

Problem 3.

a.

$$\vec{\nabla} \cdot (\epsilon \vec{E}) = 0 = \vec{\nabla} \cdot (n^2 \vec{E})$$
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon \frac{\partial}{\partial t} \vec{E} = \mu_0 n^2 \frac{\partial}{\partial t}$$

Rubric (4 points):

- +1 point for each correct equation, for a total of up to 4 points.
- It is okay if ϵ is used instead of n.
- Half a point is lost if $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon$ is written (it is not correct if there is more than one ϵ in the problem)

b. i. $\vec{E} = \vec{E}_0 e^{i(\omega t - kx)}$, where $\omega = ck/n$, and $n^2 = \epsilon/\epsilon_0$.

ii. $\vec{B} = \vec{B}_0 e^{i(\omega t - kx)}$.

iii. $\vec{E} = c\vec{B}/n = c\vec{B}\sqrt{\epsilon_0/\epsilon}$

Rubric ($\mathbf{1 + 1 + 2 = 4}$ points):

- For [i], +1 point if the equation is a plane wave moving in the x direction with the speed $c/n = c\sqrt{\epsilon/\epsilon_0}$. (The formulae for ω and n^2 are not necessary to get this point.)
- For [ii], +1 if the equation is a plane wave moving in the x direction.
- For [iii], +1 point for including n in the solution, and +1 point for replacing it with the right expression involving ϵ. (Or just +2 points for just giving the correct solution–no partial work needs to be shown for full points on this problem.)

c. ω_0 is the characteristic frequency of the material, so it is of the order of c/d, where d is the interatomic spacing, typically around an Angstrom (10^{-10} m). Therefore

$$\omega_0 \approx 3 \times 10^8/10^{-10} \approx 3 \times 10^{18} \text{ Hz} \tag{0.7}$$

And γ is the coefficient of friction for the spring.

Rubric (5 points):

- +2 points for the definition of ω_0 as a characteristic frequency.
- +1 point for the definition of γ as a friction coefficient.
- +1 point for an explanation of how to get ω_0.
- +1 point for a good order-of-magnitude guess for ω_0. (Not awarded if units are omitted.)

d.

$$\omega = \frac{ck}{n} = \frac{ck}{\sqrt{1 + (Nq_e^2/\epsilon_0 m_e)/(\omega_0^2 - \omega^2 + i\omega\gamma)}} \tag{0.8}$$

Rubric (2 points):

- +1 point for $\omega = ck/n$
- +1 point for substituting n (and not n^2, for example)

e. In the limit that $\omega \ll \gamma$,

$$n^2 \approx 1 + \frac{Nq_e^2}{i\epsilon_0 m_e \omega\gamma} = 1 - i\frac{Nq_e^2}{\epsilon_0 m_e \omega\gamma} \tag{0.9}$$

Since $\sqrt{1+x} \approx 1 + x/2$,

$$n \approx 1 - i\frac{Nq_e^2}{2\epsilon_0 m_e \omega\gamma} \tag{0.10}$$

Because the index of refraction is complex and $\omega = ck/n$ is real, the wavenumber is complex: we can write a general complex number as $k = k' - ik''$. So the electric field decays:

$$\vec{E} = \vec{E}_0 e^{i\omega t - i(k' - ik'')x} = \vec{E}_0 e^{-k''x} e^{i(\omega t - k'x)} \tag{0.11}$$

In terms of ω, we have $k = n\omega/c$. Plugging things in, we get $k' = \omega/c$ and $k'' = Nq_e^2/2\epsilon_0 m_e c\gamma$.

Rubric (5 points):

- +1 point for $\omega_0 = 0$.
- +1 point for attempt to find k from n
- +2 point for approximating n correctly. (These 2 points are not awarded if the approximation used does not satisfy $n \approx 1$)
- +1 points for plugging it in to $\vec{E}$ and writing down a solution with manifest exponential decay.

f. Those fields fall off exponentially like $e^{-k''x}$. Therefore we expect that around the distance $s = 1/k''$, the field will have fallen off by a factor of $1/e$, which is appreciable. So a valid answer is

$$s = \frac{1}{k''} = \frac{2\epsilon_0 m_e c\gamma}{Nq_e^2}\ . \tag{0.12}$$

Rubric (5 points):

- +2 points for pointing out exponential decay
- +3 points for an answer with the right units. Only +1 if the answer has an algebraic mistake and has the wrong units. Only +2 if the answer has an algebraic mistake but has the right units.
- In this problem, it is acceptable to use a different definition of "appreciable" than $1/e$. The above rubric still applies.