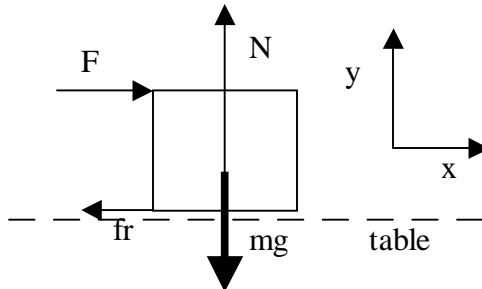


Final Prof Lanzara (Lecture 200)

Problem 1



a) The only external force acting on a block-table system is a horizontal force  $F$ .  
 Note: friction is an internal force for the block-table system.  
 External work comes from external force.

$$W_{\text{external}} = F_{\parallel} x = Fx$$

b) Energy dissipated by friction comes from work done by friction.

$$W_{\text{fr}} = \text{Energy lost}$$

$$f_r = m N$$

$$F_{y,\text{net}} = N - mg = 0, \text{ since the block is not accelerating in the } y \text{ - direction.}$$

$$f_r = m g$$

The force of friction acts over distance  $x$ .

$$W_{\text{fr}} = -m g x$$

$$\text{Energy lost} = -m g x$$

c) Work-Kinetic energy theorem can be used to solve this part.

$$W_{\text{box}} = \text{K.E.}$$

$$W_{\text{box}} = (\text{Total force acting on the box}) \cdot x = (F - f_r) \cdot x = (F - m g)x$$

$$\text{K.E.} = \frac{1}{2} m v_{\text{final}}^2 - \frac{1}{2} m v_{\text{initial}}^2 = \frac{1}{2} m v_{\text{final}}^2, \text{ since } v_{\text{initial}} = 0$$

$$\frac{1}{2} m v_{\text{final}}^2 = (F - m g)x$$

$$v_{\text{final}} = \sqrt{\frac{2(F - m g)x}{m}}$$

**Problem 2.** (Physics 7A FINAL-SECTION 2, Lanzara)

(a) The inertia moment for the hoop with radius  $R = 0.08m$  and mass  $m = 0.18kg$  becomes

$$I = mR^2 = 0.00152kgm^2$$

. Since there are two external forces acting on the hoop,  $T$  (tension, upward) and  $F_g = mg$  (gravity, downward), the equation of motion for the vertical motion reads

$$ma = T - mg \quad (1)$$

and the equation of motion for the rotation becomes

$$I\alpha = RT \sin(-90^\circ). \quad (2)$$

Because this is a rolling without slipping, the linear acceleration and angular acceleration are related by

$$a = R\alpha$$

, we can eliminate  $\alpha$  from the equation (2) and solve for tension  $T$  by eliminating  $a$ ,

$$T = mg + ma = mg - T \quad \rightarrow \quad T = \frac{mg}{2} = 0.882N. \quad (3)$$

(b) From the equations of motion found above, we have  $a = -\frac{g}{2}$  and it is a constant acceleration motion. Thus, we can use the formula

$$y = y_0 + v_0t + \frac{1}{2}at^2$$

and for  $y = 0.75m$  down from the initial position, time  $t$  becomes

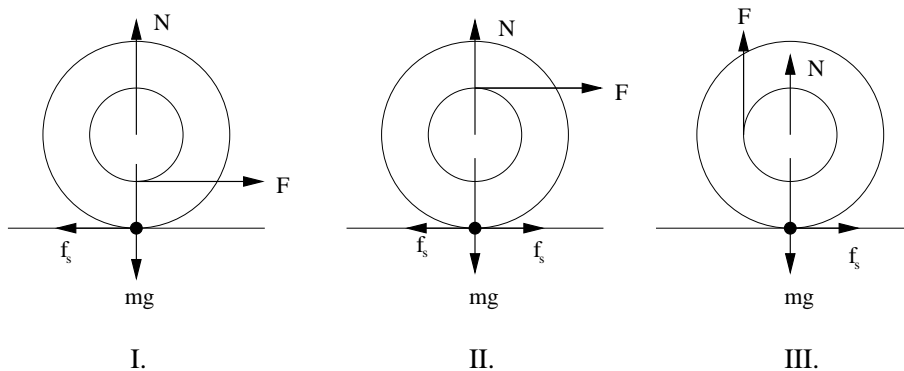
$$t = \sqrt{\frac{2(y_0 - y)}{-a}} = 0.553sec. \quad (4)$$

where  $y_0 = 0$  and  $v_0 = 0$ .

(c) For the rolling without slipping,  $v = R\omega$  and therefore

$$v = at = R\omega \quad \rightarrow \quad \omega = \frac{gt}{2R} = 33.87rad/sec.$$

(d) See figure 1. For the rolling without slipping,  $a = R\alpha$ :



There are two important points which we can choose as rotation axis, the center of the yo-yo and the contact point. The directions of rotation must be consistent for either choice of rotation and we will use this fact.

I. When we choose the contact point as the rotation axis,  $F$  is the only force that produces non-zero torque and it is clockwise.

If we choose the center of yo-yo as the rotation axis,  $F$  produces counterclockwise torque and therefore the friction  $f_s$  has to produce even larger torque clockwise. This tells that  $f_s$  must be directed to the left.

II. The direction of friction depends on the dimension of yo-yo: if  $mR > I$  then,  $f_s$  is to the right and vice versa.

III. If we take the contact point as the rotation axis,  $F$  is the only force that produces clockwise torque and this tells that  $a > 0$ . And there is only one horizontal force, friction  $f_s$  and this must be directed to the right.

(e) It rolls for all three cases because  $F$  produces clockwise torque which is the only torque when we choose the contact point as the rotation axis.



# Problem 4 - Lanzara Section 2

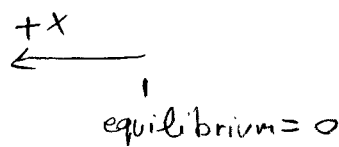
a) Simple harmonic Motion Amplitude:  $d$   $\omega = \sqrt{\frac{k}{m}}$

By the coordinate system shown:

$$x(t) = d \cos(\omega t)$$

$$v(t) = -\omega d \sin(\omega t)$$

where  $\omega = \sqrt{\frac{k}{m}}$



$$b) T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}$$

$$c) F = ma \Rightarrow -kx - bv = ma$$

$$\text{or } m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$$d) P_0 = P_1 \Rightarrow m v_0 = 2m v_1 \Rightarrow |v_1| = \frac{|v_0|}{2} \text{ to the right}$$

To find new amplitude:  $E_0 = E_1$

$$\Rightarrow \frac{1}{2} k d^2 + \frac{1}{2} (2m) \underbrace{v_1^2}_{\left(\frac{v_0}{2}\right)^2} = \frac{1}{2} k A^2$$

$$\Rightarrow A = \sqrt{d^2 + \frac{1}{2} \frac{m}{k} v_0^2}$$

∴ New Period:  $\omega_{\text{new}} = \sqrt{\frac{k}{2m}} \quad T = \frac{2\pi}{\omega}$

$$\Rightarrow T_{\text{new}} = 2\pi \sqrt{\frac{2m}{k}}$$

Prob 4 (cont)

e) Energy lost:  $E_{\text{lost}} = k \cdot E_f - k \cdot E_i = \frac{1}{2} (2m) v_1^2 - \frac{1}{2} m v_0^2$

$$= \frac{1}{2} 2m \left( \frac{v_0}{2} \right)^2 - \frac{1}{2} m v_0^2 = -\frac{1}{4} m v_0^2$$

$$\Rightarrow \boxed{|E_{\text{lost}}| = \frac{1}{4} m v_0^2}$$

f) Now at the time of collision  $m_A$  is at equilibrium point which means it has its maximum speed.

By part (a) this speed is  $d\omega = d\sqrt{\frac{k}{m}}$  to the right. Let call this  $v_A$ :

$$P_0 = P_1 \Rightarrow m(v_0 + d\sqrt{\frac{k}{m}}) = 2m v_1$$

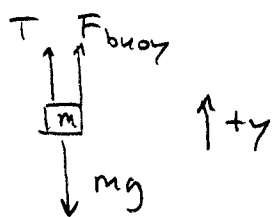
$$\Rightarrow \boxed{v_1 = \frac{1}{2} (v_0 + d\sqrt{\frac{k}{m}})} \text{ to the right}$$

Energy lost:  $E_{\text{lost}} = k \cdot E_f - k \cdot E_i$

$$= \frac{1}{2} (2m) v_1^2 - \frac{1}{2} m v_0^2$$

$$\boxed{= \frac{1}{2} m \left[ \frac{1}{2} (v_0 + d\sqrt{\frac{k}{m}})^2 - v_0^2 \right] - \frac{1}{2} k d^2}$$

# Problem 5



$$(a) \sum F_y = F_{buoy} + T - mg = 0 \quad F_{buoy} = \rho V g$$

$$\rho V g + T - mg = 0$$

$$\boxed{V = \frac{mg - T}{\rho g}}$$

$$(b) \sum F_y = F_{buoy} + T_b - mg = m(-g/3) \quad \text{* Note: } T_b \text{ not the same } T \text{ in (a)}$$

$$T_b = \frac{4}{3} mg - F_{buoy} = \frac{4}{3} mg - \rho g V \quad \text{use } V \text{ from (a)}$$

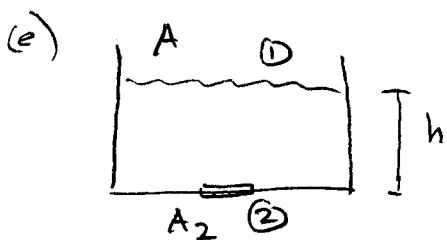
$$\boxed{T_b = T + \frac{1}{3} mg}$$

$$(c) F_{buoy} + T_c - mg = m(-g/3)$$

$$\boxed{T_c = T - \frac{1}{3} mg}$$

$$(d) F_{buoy} + T_d - mg = -mg$$

$$T_d = -F_{buoy} \Rightarrow \text{since tension can't be negative in the rope, we know } \boxed{T_d = 0}$$



$$(i) \rho g h + \frac{1}{2} \rho v_1^2 = \frac{1}{2} \rho v_2^2$$

$$2gh = v_2^2 - v_1^2$$

$$(ii) A v_1 = A_2 v_2$$

$$v_2 = \frac{A}{A_2} v_1$$

$$(iii) v_1 = -\frac{dh}{dt} \quad \text{note sign}$$

$$\text{plug (ii) into (i)} \Rightarrow 2gh = v_1^2 \left( \frac{A^2}{A_2^2} - 1 \right)$$

$$\text{simplify and plug in (iii)} \Rightarrow -\frac{dh}{dt} = \sqrt{\frac{2g A_2^2 h}{A^2 - A_2^2}}$$

$$\text{let } k = \sqrt{\frac{2g A_2^2}{A^2 - A_2^2}} \Rightarrow -\frac{dh}{dh} = k dt$$

integrate w/ initial height  $h_0$  and final time  $t_f$

$$-\int_{h_0}^0 \frac{dh}{\sqrt{h}} = \int_0^{t_f} k dt$$

$$-2\sqrt{h} \Big|_0^{h_0} = kt \Big|_0^{t_f}$$

$$\boxed{t_f = \sqrt{\frac{2h}{g} \left( \frac{A^2 - A_2^2}{A_2^2} \right)}}$$