 Test Number - B

UNIVERSITY OF CALIFORNIA
COLLEGE OF ENGINEERING

E7: INTRODUCTION TO COMPUTER PROGRAMMING FOR SCIENTISTS AND ENGINEERS

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[25 points ~ 35 minutes]

Question	Points
Part I	5
Part II	4
Part III	5
Part IV	6
Part V	5
TOTAL	**25**

Notes:

1. Your exam should have 9 pages. Check this before you begin.
2. Before you leave the exam hall, make sure your name has been cross-verified and marked on the class list by atleast one of the GSIs while you are submitting your exam. If not, your exam will not be graded.
3. You may use your notes or laptop on this examination as necessary provided that you do not impede those sitting next to you. No other electronic devices are permitted.
4. Use a #2 pencil and a green scantron sheet to record your answers. Mark your solution to each question on the corresponding space on your scantron. There is one correct answer for each question. Multiple markings, incomplete markings, or stray marks will cause your solution to be marked incorrect.
5. Please write your **name, subject, date, test number, student ID number**, and **discussion and lab section** on your scantron for identification purposes.
6. You may NOT leave the exam room before the exam ends.

Part I:

Questions 1 – 5: Computer Representation of Numbers

1. We want to write a Matlab function which takes a number z as an input and computes and returns the value of s, e and f needed to represent z in the base 10 normalization form: $z = (-1)^s \times 10^e \times f$.
   Note: $s \in \{0,1\}$, e must be an integer and f must be a fraction.

   Which of the following Matlab expressions evaluates to s?:
   A. `s = sign(z)`
   B. `s = logical(sign(z)-1)`
   C. `s = (sign(z) + 1)/2`
   D. None of the above

2. For the same problem above, what is an adequate Matlab expression for finding e?
   A. `e = round(log10(abs(z)))`
   B. `e = floor(ln(abs(z)))`
   C. `e = round(ln(abs(z)))`
   D. `e = floor(log10(abs(z)))`

3. If we limit the precision of f in $(-1)^s \times 10^e \times f$ to three decimal places, how many numbers can we represent in the interval [10, 100)?
   A. 8900
   B. 9000
   C. 9999
   D. 89000

Let us say there is a new 8 bit E7 standard for encoding numbers with the following rules – General Form : $(-1)^{sign} \times 2^{biasedExponent} \times$ Fraction, where the exponent is biased. The leading number of the fractional part after converting to general form is always 1 and always dropped. ***sign***: represented by 1st bit (0 or 1), **biasedExponent**: represented by 3 bits, ***Fraction***: represented by the remaining 4 bits.

4. What will be the result of the operation (2^5 + 1) - 2^5 ?
   A. 0
   B. 1
   C. 3
   D. 2^5

5. What will be the result of this operation (2^5 - 2^5)+ 2 ?
   A. 1
   B. 0
   C. 2
   D. None of the above

Part II:

Questions 6 - 9: Computational Complexity

6. Which of the following statements is correct about the worst-case time complexity of a program?
   A. It is a Von-Neumann machine independent property.
   B. It is a Von-Neumann machine dependent property.
   C. It is the relationship between the program's running time and memory consumption.
   D. None of the above.

7. Which of the following statements is NOT correct about time complexity? Easy
   A. Linear time is better than quadratic time
   B. Constant time is better than linear time.
   C. Linear time is better than log time
   D. Log time is better than quadratic time

8. The following Matlab function does a ternary search (splitting search interval into 3 equal parts) for a value in a sorted array.

```
function [ index ] = tSearch(array, value)
split1 = ceil(length(array)/3);
split2 = ceil(2*length(array)/3);

if array(split1) == value
    index = split1;
elseif array(split1) > value
    index = tSearch(array(1:split1-1),value);
elseif array(split2) == value
    index = split2;
elseif array(split2) > value
    index = split1 + tSearch(array(split1+1:split2-1),value);
else
    index = split2 + tSearch(array(split2+1:length(array)),value);

end
```

Let n be the length of the array input to tSearch.m. The time complexity of the function tSearch.m is -
   A. O(log(n)) but not O(n)
   B. $O(\log_3(n))$ but not O(log(n))
   C. $O(\log_2(n))$ but not $O(\log_3(n))$
   D. None of the above

9. The following Matlab function takes two equal length vectors xVector and yVector and computes the differential of yVector with respect to xVector using backward difference method.

```
function [yp] = differ(xVector, yVector)
for i =2:length(yVector)
 der(i-1) = (yVector(i)-yVector(i-1))/(xVector(i)-xVector(i-1));
end
end
```

Let n be the length of the vectors input to differ.m. The time complexity of the function differ.m is -

A. O(n^2).
B. O(1) always.
C. It can't be determined.
D. O(n).

Part III:

Questions 10 - 14: System of Linear Equations and Root finding

10. If the following system of equations is re-written in the matrix form AX = b, where X = [x; y; z], what will be A and b in Matlab?

$2x + 3y + 9z = 5$
$3x + 5y + 8z = 2$
$x + 3y + z = 1$

A. A = [2 3 1; 3 5 3; 9 8 1] and b = [5; 2; 1]
B. A = [2 3 9; 3 5 8; 1 3 1] and b = [5 2 1]
C. This system has no matrix form
D. None of the above

11. If M = [1, 2, 3 ; 4, 5, 6 ; 3, 6, 9] and N = [1 ; 5 ; 3], which of the following will produce a correct solution to the system of equations MX=N in Matlab ?

A. `inv(M)*N`
B. `mldivide(M,N)`
C. `pinv(M)*N`
D. `M\N`

12. Consider the following equation : Ax = b
where, A is a 2x2 matrix and b is a 2x1 matrix. The rank of A is 1 and the rank of [A b] is 2. Which statement is true?

A. The equation has infinite solutions
B. The equations has one unique solution
C. The equation has no solution ✓
D. None of the above

13. Consider the function f(x) = 5-2x
What is the value of the approximate root of f(x) after 3 iterations using the bisection method on the interval [0 7] ? (Note: approximate root after 0^{th} iteration is (0+7)/2 = 3.5)

A. 0
B. 1.75
C. 2.525
D. 2.1875

14. Consider the function $f(x) = (x-2)^2$
The following Matlab function computes the root of f(x) by Newton Raphson method.

```
function [ root ] = nr( guess )
x = guess;
iter = 0;
while true
    d = ((x-2)^2)/(2*x-4);
    if d ==0
        break;
    end
    x = x - d;
    iter = iter + 1;
    fprintf('Iteration %d: x=%.20f\n', iter, x);
end
root = x;
end
```

Which of the following is true about the function for an initial guess, x=10?

A. nr(10) will not halt
B. nr(10) will halt after 1 interation
C. nr(10) will halt after 2 interations
D. None of the above

Part IV:

Questions 15- 20: Least Squares Regression and Interpolation

15. When conducting polynomial interpolation, which of the following is always true about the degree of the polynomial that fits n data points exactly?

A. It must be greater than the total number of data points +1
B. It must not exceed the total number of data points – 1
C. It can be any integer.
D. None of the above

16. In using the Matlab command:
`coef = polyfit(x,y,4);`
`coef(2)` is the coefficient of which power of x?

A. x^3
B. x^2
C. x^1
D. x^0

17. Consider the following Matlab function that takes two vectors `xdata` and `ydata` of equal length and returns the least squares estimate `ye` of the given `ydata`.

```
function [ye] = lsq(xdata, ydata)
   co = polyfit(xdata, ydata, 1);
   ye = co(1)*xdata + co(2);
end
```

Which of the following is true?

A. `mean(ye)*mean(ydata)` will return 0
B. `mean(ye) - mean(ydata)` will return -1
C. `mean(ye)/mean(ydata)` will return 1
D. None of the above

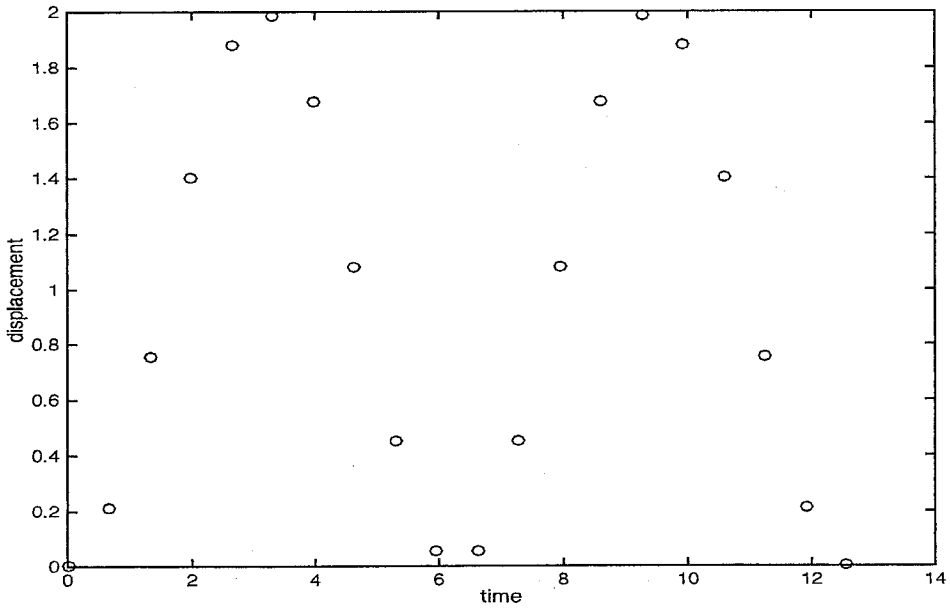
18. Which of the following is true about cubic spline interpolation on a set of *n* data points?

A. Only the slopes of two adjacent cubics are required to be equal.
B. It fits a (n-1) degree polynomial through the n data points.
C. It fits (n-1) cubics through the n data points.
D. All of the Above

19. Vector X contains numbers representing distance (miles) from the city center. Vector Y contains numbers representing the number of customers for a company at every corresponding distance in vector X. Which expression predicts the number of customers for that company at 30 miles from the center based on a quadratic regression?

A. polyfit (X, Y, 30)
B. polyval (X, Y, 30)
C. polyval(polyfit(X,Y,2), 30)
D. polyfit(polyval(X,Y,2), 30)

20. You are given the following plot generated from displacement data y collected at different times t.



You want to find the displacement at any given time T. There are two possible options available to you -

1. d_t = interp1(t,y,T)
2. d_t = spline(t,y,T)

Which of the following statements is correct?

A. 1 will always underestimate compared to 2
B. 1 will always be greater than 2
C. 1 will sometimes overestimate compared to 2
D. 1 will always be less than 2

Part V:

Questions 21 - 25: Numerical Differentiation and Integration

21. What is the order of accuracy of forward difference approximation of a differential?

    A. 4^{th} order
    B. 2^{nd} order
    C. 1^{st} order
    D. None of the above

22. y is a vector of displacement of an object measured at times given in vector t. The plot of this data was shown in Problem 20. Suppose the velocity of the object is computed by:

```
>> y1 = diff(y)./diff(t);
```

    Which of the following will plot the acceleration of the object?

    A. `plot(t(3:end), diff(y1))`
    B. `plot(t(3:end), diff(diff(y))./diff(t(2:end)))`
    C. `plot(t(3:end), diff(y1)./diff(t(2:end)))`
    D. `plot(t(3:end), diff(y1)./diff(t))`

23. Now assume in the above example, the displacement vector had 1000 values. We used forward difference and backward difference methods to compute the velocity vectors, Vf and Vb respectively in Matlab. You are interested in finding the velocity of the object at the last time instant. What expression will give you this value?

    A. Vf(999)
    B. Vb(999)
    C. Vf(1000)
    D. None of the above

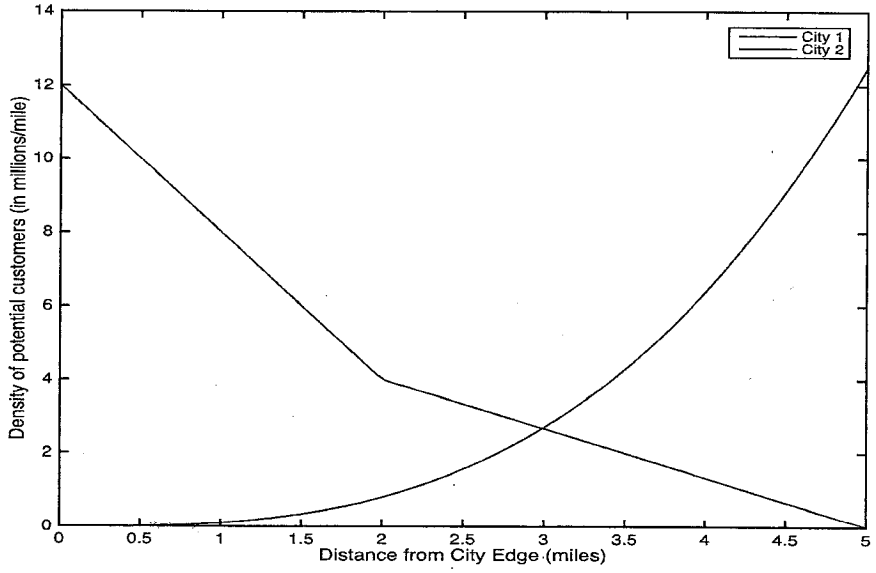
A food distributor needs to supply at least 1 food item per 1 potential customer. The number of potential customers varies with distance and is represented by the following 2 equations. Every equation represents a different city:

City 1: $Y(x) = (x^3)/10$ $0 <= x <= 5$ (miles)

City 2: $Y(x) = 12 - 4x$ $0 <= x <= 2$ (miles)

$= 4 - 4/3(x-2)$ $2 < x <= 5$ (miles)

where Y(x) is the density of potential customers (in millions/mile) at distance x from the city edge. The density of customers given by the above equations is plotted below.



24. To calculate the quantity of food to be supplied most accurately, which is the best method the distributor can use?

A. Trapezoidal Rule for City 1 and Simpson's Rule for City 2 (with 100 subintervals)
B. Simpson's Rule for both City 1 and City 2 (with 100 subintervals)
C. Trapezoidal Rule for both City 1 and City 2 (with 50 subintervals)
D. Simpson's Rule for City 1 and Trapezoidal Rule for City 2 (with 50 subintervals)

25. Suppose the distributor decides to use Trapezoidal Rule for City 1, what can you say about the relation between the error (if any) in the quantity of food calculated and the number of subintervals N chosen for the integration?

A. The error is inversely proportional to N^2
B. The error is directly proportional to N^2
C. The error is directly proportional to N^4
D. Tricky! Tricky! There is actually no error! So it is obviously independent of N.