Math 54, **Midterm III**, F.Rezakhanlou

Each question should be answered directly. Use the back of these sheets if necessary. Justify your assertions; include detailed explanation, and show your work. No aid (including calculators) are allowed.

Your Name:

Your GSI's Name:

Your Section:

- **1.** Solve $y^{(4)} + 8y'' + 16y = 0$, subject to initial conditions $y(0) = y'(0) = y''(0) = 0,\ y'''(0) = 1$.

- **2.** Find the general solution of $\mathbf{x}'(t) = \begin{bmatrix} 1 & 0 & -2 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \mathbf{x}(t)$.

- **3.** Find the general solution of $y'' + y = 2t + 1 + (\cos t)^{-2}$.

- **4. (True - False)**

For each of the questions below, indicate if the statement is **true** or **false**. If true, **justify** (give a brief explanation or quote a relevant theorem from the course), and if false, give a counter-example or explain.

(a) If $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ are three solutions of $\mathbf{x}'(t) = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 2 \\ 2 & 5 & -1 \end{bmatrix} \mathbf{x}(t)$, then the Wronskian of $W[\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3](t)$ is constant.

(b) There is at most one solution to the initial value problem

$$y''' + e^t y'' + y(t) = t^2,$$
$$y(0) = y'(0) = 1.$$

(c) There exist continuous functions $p(t)$ and $q(t)$ such that $y_1(t) = \sin t$ and $y_2(t) = t \sin t$ solve the differential equation $y'' + py' + qy = 0$.

(d) There exist 5 linearly independent solutions to the differential equation

$$y''' + t^2 y'' + (\cos t)y = 0.$$

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