Math 54, **Midterm II**, F.Rezakhanlou

Each question should be answered directly. Use the back of these sheets if necessary. Justify your assertions; include detailed explanation, and show your work. No aid (including calculators) are allowed.

Your Name:

Your GSI's Name:

Your Section:

- **1.** Find a matrix Q that orthogonally diagonalize the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$.

- **2.** Let S be the span of $\mathbf{w}_1 = (0,0,1,1)$, $\mathbf{w}_2 = (2,-2,5,-4)$, $\mathbf{w}_3 = (2,-2,0,0)$. Use Gram-Schmidt process to find an orthogonal basis for S. What is $proj_S\mathbf{a}$ for $\mathbf{a} = (1,0,0,0)$?

• **3.** (a) For matrices $A_1 = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$, $A_2 = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$, define

$$< A_1, A_2 >= \lambda_1 a_1 a_2 + \lambda_2 b_1 b_2 + \lambda_3 c_1 c_2 + \lambda_4 d_1 d_2 \ .$$

Show $< A_1, A_2 >$ defines an inner product on the space of 2×2 matrices $M_{2,2}$, *if and only if* $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are positive.

(b) Let $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and let $\lambda_1 = \lambda_2 = 1$, $\lambda_3 = \lambda_4 = 2$ in part (a).
Define $V = \{A| < A, B >= 0\}$. Show that V is a subspace of $M_{2,2}$.
What is the dimension of V? Find a basis for V.

- **4. (True - False)**

  For each of the questions below, indicate if the statement is **true** or **false**. If true, **justify** (give a brief explanation or quote a relevant theorem from the course), and if false, give a counter-example or explain.

  (a) The matrices A and A^T have the same eigenvalues.

  (b) If A is a square matrix, then $\|Ax\| = \|A^T x\|$.

  (c) If A is an orthogonal matrix, then the rows of A form an orthonormal basis.

  (d) If A is a matrix with eigenvalues $1, 2, 3$, then $(A + 2I)^{-1}$ has eigenvalues $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$.

  (e) There exist vectors $\mathbf{a}$ and $\mathbf{b}$ in $\mathbb{R}^5$ such that the linear transformation $T(\mathbf{x}) = (\mathbf{x}\cdot\mathbf{a})\mathbf{b}$ is of rank 3.

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