Math 54, F.Rezakhanlou
First Midterm, Sept. 30, 1996

Each question should be answered directly. Use the back of these sheets if necessary.
Justify your assertions; include detailed explanation, and show your work. Closed book exam,
no sheet of notes and no calculator.

1. Let $A = \begin{bmatrix} 0 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \beta & 3 & 2 & 3 \\ 1 & 5 & 1 & 1 \end{bmatrix}$.

(a) *(8 pts)*

Calculate the determinant of A.

(b) *(2 pts)*

For what values of β is A an invertible matrix?

2. Let A be as in the previous problem.

   (a) *(5 pts)*
        What is the reduced echelon form of A when A is invertible?

   (b) *(15 pts)*
        Find an *echelon* form of A when A is not invertible.

   (c) *(10 pts)*
        When A is not invertible, solve the equation $A\vec{x} = 0$.

3. *(15 pts)*

Let A be a 4×4 matrix with all ones: $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

(a) Show $A^2 = 4A$.

(b) Let $B = A + 2I$. Show $8B - B^2 - 12I = 0$.

(c) Show that B is invertible and find B^{-1}.

4. **True - False** *(20 points)*

For each of the questions below, indicate if the statement is **true** or **false**. If true, **justify** (give a brief explanation or quote a relevant theorem from the course), and if false, give a counter-example or explain.

(a) If A and B are two square matrices, then $(A + B)(A - B) = A^2 - B^2$.

(b) Suppose A and B are two square matrices and **a** is a vector such that $A\mathbf{a} = 2\mathbf{a}$ and $B\mathbf{a} = 3\mathbf{a}$. Then $AB\mathbf{a} = 6\mathbf{a}$.

(c) If $\det A = 2$, then $\det A^{-1} = \frac{1}{2}$.

(d) If $\det A = 2$, then $\det(5A) = 10$.

(e) If A is a 3×3 matrix with all ones, then $\det A = 0$.