Problem 1 Solutions

(a) True or False: 0.5 point each

FALSE (i) The heat capacity of a body is the amount of heat that it can store at a given temperature. *Heat capacity tells you how much the temperature changes when heat is added.*

FALSE (ii) If the absolute temperature of a body triples, the rate at which it radiates energy increases by a factor of 9. *Remember:* $P \propto T^4$*, so the increase would be by a factor of 81.*

TRUE (iii) When a system goes from equilibrium state 1 to state 2, the change in the internal energy is the same for all processes. *Internal energy is a state function.*

TRUE (iv) The internal energy of a given amount of an ideal gas at equilibrium depends only on its absolute temperature. *This is a main result of the equipartition theorem.*

TRUE (v) For any material that expands when heated, C_P is greater than C_V. *This happens because it costs energy to do the work of expansion.*

TRUE (vi) Pressure and Temperature are state variables. *They only depend on the current state of the gas, not on a process.*

TRUE (vii) Thermal equilibrium is when the thermodynamic variables are constant in time and throughout the system.

This is the definition of thermal equilibrium. You may argue that we know of processes that are in thermal equilibrium but change over time. This is only an approximation that becomes valid when the process is done slowly (quasistatically). In reality, the gas is never truly in equilibrium if things are changing.

TRUE (viii) The Carnot efficiency changes by the same amount from either increasing the hotter temperature reservoir by 10% or decreasing the lower reservoir temperature by 10% to within 1%. *You can note that* $\frac{1}{1-0.1} \approx (1+0.1)$ *to within about 1% accuracy. It's just the Taylor expansion.*

TRUE (ix) A good emitter of radiation is a good absorber of radiation.

TRUE (x) According to the principle of equipartition, energy is shared equally among the active degrees of freedom in an amount of $\frac{1}{2}kT$ on average. *This is simply a summary of the equipartition theorem.*

Part (b): 1 point each

(i) A scuba diver releases a spherical air bubble 3-cm diameter at a depth of 14-m in water at 298 K. Approximately what is the air bubble's diameter just as it reaches the surface, if the temperature stays constant. Take the density of water to be 1000 kg/m^3.
(A) 3 cm.
(B) 4 cm.
(C) 5 cm..
(D) 6 cm
(E) 7 cm.
Answer: (B)
Why: $PV = P_0 V_0 = nkT$ We need to remember how pressure increses with depth. If you forgot, you could probably argue for $P = P_0 + \rho g h$. Now, we just plug in numbers to get the new volume and then the new diameter. $P_0 = 15m \times 1000 \times 9.8 + 1.013^5$N/m^2 $= 2.48300 N/m^2$ $P = 1.013 \times 10^5 N/m^2$ $V = V_0 \times 2.48/1.013 = 2.45114 V_0 = 2.45114\pi(3cm/2)^3/4 = \pi r^3/4$ $d = 2r = 3.0 * (2.45114)^{1/3} = 4.04492cm$

(ii) You are on a trip overseas and it seems very cold. You use a local thermometer to see how cold it really is It reads -40°C, what temperature is that in ° F? (A) -104° F.
(B) -60° F.
(C) -40° F.
(D) -20° F.
(E) -10° F..
Answer: (C) -40° F..
Why: $X°F = 9Y°C/5 + 32°F = 9 \times -40/5 - 32 = -40°F$. If you forgot the formula, you can figure it out using two data points: $100°C = 212°F$ and $0°C = 32°F$.

(iii) A 1 meter radius balloon containing an ideal gas is cooled from 400 K to 300 K. How much does its volume and surface area change assuming the external pressure is constant?
(A) decrease by 33% and 21%.
(B) decrease by 25% and 17%.
(C) no change.
(D) increase by by 25% and 17%.
(E) increase by 33% and 21%.
Answer: (B) decrease by 25% and 17%.
We use the ideal gas law $PV = nRT$ which we rewrite as $V = (nR/P)T$. That means the ratio of volumes is $V_{300}/V_{400} = 300/400 = 0.75$ which is a 25% decrease in volume. The area is $4\pi R^2$ and the volume is $4\pi/3\ R^3$ so that the area ratios will be the 2/3 power of the volume ratios.

(iv) A 1 meter radius glass sphere is heated by 100°C. About how much does its surface area increase, given $\alpha = 9 \times 10^{-6}/°C$?
(A) increase by 3 cm^2.
(B) increase by 50 cm^2.
(C) increase by 100 cm^2.

(D) increase by 200 cm^2
(E) increase by 300 cm^2
Answer: (D) increase by 200 cm^2
Why: Radius will increase by
$R = R_o(1 + \alpha\Delta T) = 1m * (1 + 9 \times 10^{-6} \times 100) = 1.0009m$
Change in area is $A - A_o = 4\pi(R^2 - R_o^2) = 4\pi(1.0009^2 - 1)m^2 = 0.0226m^2 = 226cm^2$. In the same way that the volume coefficient of expansion is 3 times bigger than linear expansion, the *area* coefficient will be about twice the linear coefficient.

(v) At standard temperature (273 K) and pressure (1 atm) how many air molecules are there per cubic cm?
(A) 269 molecules/cm^3.
(B) 2.69×10^{19} molecules/cm^3.
(C) 2.69×10^{23} molecules/cm^3.
(D) 6.02×10^{23} molecules/cm^3.
(E) 2.69×10^{25} molecules/cm^3.
Answer: (B) 2.69×10^{19} molecules/cm^3.
Use the ideal gas law $PV = NkT$, solve for $N/V = P/kT$, and substitute in. Now, just substitute in and be sure to convert from m^3 to cm^3.

Part (c): 1 point each
(i) Two gases are mixed and in equilibrium, the ratio of their molecules rms speeds is equal to
(A) unity. The molecules have the same rms speed.
(B) the ratio of their masses.
(C) inverse ratio of the square root of their masses.
(D) inverse ratio of their masses.
(E) ratio of the square root of their masses.
Answer: (C) inverse ratio of the square root of their masses.
Why: $v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\rho}{P}}$ so that the ratio of the speeds is $v_1/v_2 = \sqrt{m_2/m_1}$. We could have figured out that formula for v_{rms} by setting $\frac{1}{2}mv_{rms}^2 = \frac{1}{2}k$

(ii) Gas molecules in equilibrium have a distribution which is **NOT**
(A) given by the Maxwell Distribution for speed.
(B) has a mean kinetic energy per degree of freedom of kT/2.
(C) symmetric in velocity and asymmetric in speed.
(D) has an rms speed that is proportional to the absolute temperature.
(E) has a lower average than rms speed.
Answer: (D) have an rms speed that is proportional to the absolute temperature.
Why: This is a little tricky: the rms speed is related to T, but not proportional. $v_{rms} = \sqrt{\frac{3kT}{m}}$

(iii) The critical point of a real gas is

(A) where the gas changes to a liquid.

(B) where the PV isotherm curve goes horizontal.

(C) the temperature which divides the regions where a gas can be made a liquid by applying pressure and where it cannot.

(D) the pressure and volume on the critical temperature isotherm curve where it goes critical.

(E) the point where gas, liquid and solid can coexist in equilibrium.

Answer: (D)the pressure and volume on the critical temperature isothem curve where it goes critical.

Why: The point where gas, liquid and solid can coexist in equilibrium is called the **triple point**.

The temperature which divides the regions where a gas can be made a liquid by applying pressure and where it cannot is called the **critical temperature**.

The PV isotherm curve goes horizontal for all temperatures below the critical temperature.

The gas can change to a liquid along a line of pressure and temperatures.

(iv) Which of these gas laws are incorrectly stated?

(A) Avogadro's Law: $V \propto n$ for constant temperature and pressure.

(B) Diver's Law $P \propto n$ for constant temperature and volume.

(C) Boyle's Law: $V \propto \frac{1}{P}$ at constant temperature.

(D) Charles's Law: $V \propto T$ for constant pressure.

(E) Gay-Lussac's law: $P \propto T$ for constant volume.

(F) Ideal Gas Law: $PV = nRT = NkT$

(G) Clausius equation of state: $P\left(\frac{V}{n} - b\right) = RT$

(H) van der Waals equaiton of state: $\left(p + \frac{a}{(V/n)^2}\right)\left(\frac{V}{n} - b\right) = RT$

(I) Dalton's law: two gases mixed together in equilibrium $P_1 + P_2 = P_{total} = (n_1 + n_2)RT/V$

(J) All of the above are correct.

Answer: (J) All of the above are correct. Answers can be found by using the Ideal Gas Law or remembering discussion in the book.

(v) What temperature is used for ideal gas law and heat engine calculations and formula:

(A) absolute temperature using Kelvin scale K.

(B) °C = Centigrade or Celsius scale.

(C) °F = Farhenheit.

(D) R = Rankine.

(E) °Reaumur

(F) any of these temperature scales.

Answer: (A) absolute temperature using Kelvin scale K.

If you care, a Rankine is an absolute temperature scale based not on Celsius (like Kelvin) but on Fahrenheit. Reaumur is an out of date temperature scale that sets 0 to the freezing point of water and 80 to boiling.

Part (d): 1 point each

(i) You have been running track races in smooth-soled shoes. During each start, you have been wasting 100 joules of energy as thermal energy because of friction between your shoes and the track. To help this situation, you purchase a pair of spiked shoes. Now when you start a race, the frictional force your feet experience from the track is increased by a factor of 5 and the shoes do not slide across the track at all. During each start, the amount of energy you now waste as thermal energy because of friction between your spiked shoes and the track is **(A) 0 joules.**

(B) 4 joules.
(C) 500 joules.
(D) 20 joules.
(E) not enough information to determine the change.
Answer: (A) 0 joules.

Why: Only sliding friction turns ordered energy into thermal energy. Since your spiked shoes prevent your feet from sliding on the track, you waste no energy at all as thermal energy due to friction between your shoes and the track. There is *static* friction, but that won't convert kinetic energy into thermal energy.

(ii) If the The total energy of a rubber ball in a box is contained in the ball's gravitational potential energy, its kinetic energy of motion, and its thermal energy. Energy can be transferred from one of these forms to another as the ball moves around. You throw the ball into the box and leave it for 10 minutes. When you return, most of the ball's energy will have

(A) turned into thermal energy.
(B) turned into gravitational potential energy.
(C) turned into kinetic energy of motion.
(D) turned into random bouncing of the ball around the box.
(E) shared equally into all forms of energy.
Answer: (A) turned into thermal energy.

We assume that this is a REAL ball. So, it will eventually stop bouncing and settle into the bottom of the box. It will have no kinetic energy, and it lost it's initial gravitational energy. All the energy had to go somewhere, so it all became thermal energy.

(iii) Which of these is **not** a latent heat?

(A) heat of fusion.
(B) heat of vaporization.
(C) heat of phase change.
(D) heat of friction.
(E) none of the above
Answer: (D) heat of friction

Latent heat is the amount of energy released in a change of phase. Heat of friction is the conversion of motional energy into heat energy.

(iv) A soft drink is rated to have 160 Calories (=160 kilocalories). What is the equivalent energy in joules?

(A) 669 joules.

(B)6.69×10^3.
(C) 6.69×10^4.
(D) 6.69×10^5.
(E) 6.69×10^6.
Answer: (D) 6.69×10^5.
Just use the conversion: 1 Calorie $= 4.18 \times 10^3$ joules. That's a lot of energy!

(v) At standard temperature (273 K) and pressure (1 atm) how many air molecules are there per cubic cm?
(A) 269 molecules/cm^3.
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(C) 2.69×10^{23} molecules/cm^3.
(D) 6.02×10^{23} molecules/cm^3.
(E) 2.69×10^{25} molecules/cm^3.
Answer: (B) 2.69×10^{19} molecules/cm^3.
This is the same question as before. We just counted it twice. If this was left blank, we assumed that your previous answer applies to this question, too.

Part (e): 1 point each

(i) Which of these is **not** a possible efficiency for a heat engine working between to heat reservoirs at 127° and 27°C?
(A) zero.
(B) 0.1
(C) 0.2
(D) 0.25
(E) 0.3
(F) all above are acceptable
Answer: (E) 0.3
The efficiency formula gives: $e = 1 - \frac{T_L}{T_H} = 1 - \frac{27+273}{127+273} = 1 - \frac{300}{400} = 0.25$. This is the Carnot efficiency, and no heat engine can exceed this efficiency. A zero efficiency is definitely possible, if it doesn't do any work.

(ii) What is the change in entropy of 1 kg of water when it is heated from 0°C to 100°C?
(A) 10 cal/K
(B) 100 cal/K
(C) 1000 ln(373/273) cal/K
(D) 1000 cal/K
(E) 1000 ln(273/373) cal/K
Answer: (C) 1000 ln(373/273) cal/K
Why: Start with the normal entropy formula. $\Delta S = \int \frac{dQ}{T} = \int \frac{mC_p dT}{T} = mC_p ln(T_f/T_i)$ $T_f = 273 + 100 = 373$K. $T_i = 273$K

(iii) The mean time between collisions of air molecules at standard temperature and pressure is roughly

(A) 10^{-20} sec
(B) 10^{-20} sec
(C) 10^{-15} sec.
(D) 10^{-10} sec.
(E) 10^{-5} sec
Answer: (D) 10^{-10} sec.

To get this answer, one has to make some crude approximations. Note, however, that we only want a rough "order of magnitude" answer. The rms speed of air molecules, O_2 and N_2, at room temperature (20°C) is about $500 m/s$ according to the text on page 470. You can figure out what it is if you didn't remember that off the top of your head. You can convert the density at STP to a rough spacing between molecules. You may remember that the mean free path is actually longer than this. So, you'll get an answer that's too low. Thus, $\Delta t \approx \Delta x / v_{rms}$. You get 7×10^{-12} seconds. Thus, we want to choose 10^{-10} seconds.

(iv) The work done by an ideal gas that adiabatically expands from pressure P_1 and volume V_2 to pressure P_1 and volume V_2 is given by the expression

(A) $P_1 V_1^\gamma - P_2 V_2^\gamma$.
(B) $(P_1 V_1 - P_2 V_2)^\gamma$.
(C) $(P_1 V_1 - P_2 V_2)/(\gamma - 1)$.
(D) $(P_1 V_1 - P_2 V_2)^{1/\gamma}$.
(E) $P_1 (V_1 - V_2)$.
Answer: (C) $(P_1 V_1 - P_2 V_2)/(\gamma - 1)$.

Why: $PV^\gamma = constant = P_1 V_1 = P_2 V_2$ $W = \int P dV = \int constant dV / v^\gamma = \frac{constant}{\gamma - 1} \left(V_2^{-\gamma+1} - V_1^{-\gamma+1} \right) = \frac{P_1 V_1^\gamma}{\gamma - 1} \left(V_2^{-\gamma+1} - V_1^{-\gamma+1} \right)$ thus $W = \frac{P_1 V_1}{\gamma - 1} \left[\left(\frac{V_1}{V_2} \right)^{\gamma - 1} - 1 \right]$. Going one step further $P_1 V_1^\gamma = P_2 V_2^\gamma$ means $P_2 V_2 = P_1 V_1 \left(\frac{V_1}{V_2} \right)^{\gamma - 1}$ so that $W = \left(P_2 V_2 - P_1 V_1 \right) / (\gamma - 1)$

That's the long way. You could also notice that $W = -\Delta E = -d/2 \Delta(NkT)$.
But, $\Delta(Nkt) = \Delta(PV)$, by the Ideal Gas Law.
But $d/2 = \frac{1}{\gamma - 1}$ (Check it!). Bring it all together and get (C).

(v) If the entropy of a substance is proportional to its absolute temperature, T, then its heat capacity must be proportional to

(A) 1/T.
(B) constant (T^0)
(C) T.
(D) T^2.
(E) T^3.
Answer: (C) T.

Why: $\Delta S = \int dQ/T = \int C dT / T \propto T$ Taking the derivative wrt T gives $C/T = (constant)$ which means $C = (constant) \cdot T$. Another way to see this is by noticing that if $S \propto T$, then the temperature had to cancel out of the integral. The only way for this to happen is if $C \propto T$

2. [15 points] You are a scientist or engineer working at the Admunsen-Scott South Pole station. Staying warm enough to survive is a priority. The ice surface and air temperature is roughly -45° C.
(a) [4 points] The area of each of your boot's bottom is about 2500 cm^2. If the boot bottoms are 3 cm thick and made of insulating plastic with thermal conductivity of about 10^5 kcal/(s*m*°C), how much heat is lost through the soles of your boots?

We can simply use the equation for heat flow through a material with area A, length l, and thermal conductivity k,

$$\frac{dQ}{dt} = \frac{kA}{l}\Delta T.$$

In this problem the parameters given are in error. The correct thermal conductivity is $k = 10^{-5} kcal/(s\text{-}m\text{-}K)$*, and a more realistic area for each boot is* 250 cm^2*. With these numbers, the heat flow is found from:*

$$k = 10^{-5}\frac{kcal}{s\text{-}m\text{-}K} \times 4.186 \times 10^3 \frac{J}{kcal} = 4.186 \times 10^{-2}\frac{J}{s\text{-}m\text{-}K}$$

$$A_{tot} = 2A_{boot} = 2 \times 250\ cm^2 = 5 \times 10^{-2} m^2$$

$$\frac{dQ}{dt} = \frac{\left(4.186 \times 10^{-2}\frac{J}{s\text{-}m\text{-}K}\right)(5 \times 10^{-2} m^2)}{.03m}(34°C - -45°C) = 5.52W$$

If you use the parameters listed on the test, your answer is $5.52 \times 10^{11} W$*, eleven orders of magnitude too large. To give an idea how much power this is, Diablo Canyon nuclear power plant puts out about 2,000 MW =* 2×10^9 *W, or less than 1 percent of the number calculated with the numbers given. Since the errors were on the test, this bodaciously huge number was accepted as correct.*

(b) [4 points] You go outside when the Sun is shining. Your effective cross-sectional area is about 0.7 m^2. What is your heat gain from the Sun?

Giancolli lists the intensity of the suns radiation as $1000W/m^2$*. You are already given your projected area to the sun, so you don't need to take into acount any* $\cos\theta$ *effect from being at the pole. Estimating an emissivity of about .5 (With a big jacket its probably worse than this):*

$$\frac{dQ}{dt} = 1000\frac{W}{m^2} \times .5 \times .7m^2 = 350W$$

Not much information was given here, so a wide range of reasonable asnwers were accepted.

(c) [2 points] Why does it matter whether the wind is blowing or not?

Convection is a very effective form of heat transfer. Air's thermal conductivity is poor, so stagnant air is a reasonably good insulator - but moving air can produce signifigant heat transfer through convection. Note that this problem was basically testing if you knew what convection is, so it was important to answer using the word "convection".

(d) [1 points] Does it matter if you are moving?

You moving through still air is the same from your frame of reference as a wind blowing towards you, so this also results in convection.

(e) [4 points] What about breathing air in and out? A typical breathing rate is roughly 6 liters per minute (about a quarter of a mole of air per minute) what heat loss rate does that correspond to? Does it matter that you breath freezes on your mask? Does that cause heat loss?

Each breath you take in cold air, and exhaust hot air. We can calculate the rate of heat loss by seeing how much heat we give to the air in changing its temperature. The temperature inside the lungs is roughly 37° C, and the lungs operate at roughly constant pressure. Since the air is predominately made up of diatomic gasses, to heat up one mole of the arctic air it takes

$$\frac{7}{2}R\Delta T = \frac{7}{2}(8.3\frac{J}{mole\text{-}K})(37^\circ C - -45^\circ C) = 2380\frac{J}{mole}$$

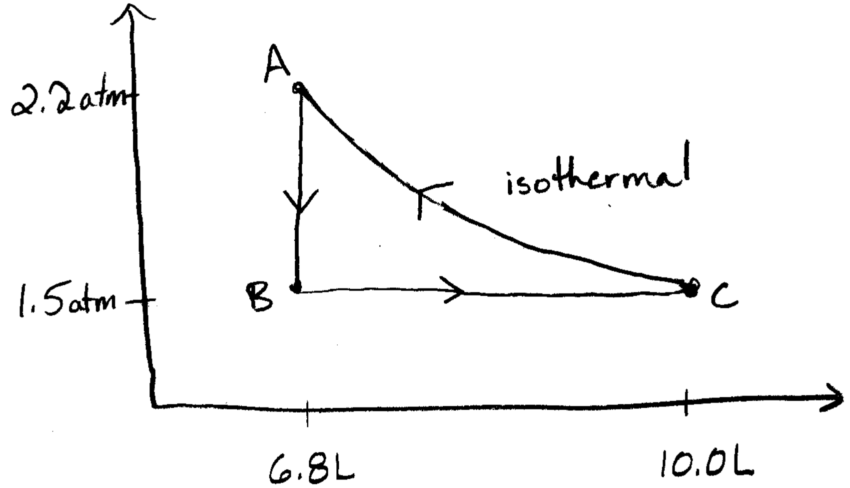
Now we are breathing at a rate of roughly .25 moles per minute. So our rate of heat loss is:

$$\frac{dQ}{dt} = .25\frac{mole}{minute}2380\frac{J}{mole} = 596\frac{J}{minute} = 9.93W$$

As for breath freezing on the mask. The heat was lost when you heated up the air in your lungs. What happens to the air after it leaves you is of no concern to you any more. Now, the fact that in your lungs the chemistry of the gas changes, and thus C_P of the gas will change while you are heating up the air in your lungs. However, this will be a small effect, as most of the input and output gas is diatomic, and our calculation is order of magnitude correct. Note that since 34° C was given as the human body temperature, students using 34° C for the temperature of the lungs and getting 9.56 W also received full credit.

3. [20 points] Consider the following three-step process cycle: Heat is allowed to flow out of an ideal monatomic gas at constant volume so that its pressure drops from 2.2 atm to 1.5 atm. Then the gas expands at constant pressure, from a volume of 6.8 L to 10.0 L, where the temperature reaches its original value. The gas then moves along the isotherm back to its starting point.

(a) [4 points] Draw a PV diagram showing the three process cycle.



(b) [4 points] Calculate the total work done by the gas in the process cycle.

$W_{AB} = 0$ (no ΔV)

$W_{BC} = P_B(V_c - V_B) = 1.5\text{atm}(10.0\text{L} - 6.8\text{L}) = 4.8\text{ atm}\cdot\text{L}$

* conversion factor $\frac{1\text{J}}{1\text{atm}\cdot\text{L}} = \frac{1.013\cdot10^5\text{Pa}}{1\text{atm}} \cdot \frac{1000\text{mL}}{1\text{L}} \cdot \frac{1\text{cm}^3}{1\text{mL}} \cdot \frac{1\text{m}^3}{10^6\text{cm}^3} = 1.013\cdot10^2$

so $W_{BC} = 4.8\cdot1.013\cdot10^2\text{J} = \underline{486.2\text{J}}$

$W_{CA} = \int P\,dV = \int_{10}^{6.8} \frac{nkT}{V}\,dV = nkT\ln\left(\frac{6.8}{10.0}\right) = P_cV_c\ln\left(\frac{6.8}{10}\right)$

$= 1.5\text{atm}\cdot10\text{L}\cdot\ln(.68) = -5.78\text{atmL}$

$= -5.78\cdot1.013\cdot10^2\text{ J} = \underline{-586.0\text{J}}$

$W_{TOT} = W_{BC} + W_{CA} = 486.2\text{J} - 586.0\text{J} = \boxed{-99.8\text{J}}$

(c) [4 points] Calculate the change in internal energy of the gas in the first two steps (processes) of the cycle.

$$\Delta U_{TOT} = 0 = \Delta U_{AB} + \Delta U_{BC} + \underset{\text{isotherm}}{\cancel{\Delta U_{CA}}^{\to 0}} = 0 \quad \text{so} \quad \boxed{\Delta U_{AC} = 0}$$

OR Can also calculate

$$\Delta U_{ab} = \frac{3}{2} NK\Delta T = \frac{3}{2}NK\left(\frac{P_bV_b}{NK} - \frac{P_aV_a}{NK}\right) = \frac{3}{2}(1.5\cdot 6.8 - 2.2\cdot 6.8)$$

$$= -7.14 \text{ atm L} \cdot 1.013\cdot 10^{2}\ \text{J/atm L} = \boxed{-723\ \text{J} = \Delta U_{ab}}$$

$$\Delta U_{bc} = -\Delta U_{ab} = \boxed{723\ \text{J}}$$

(d) [4 points] What is heat flow into or out of the gas on each of the three steps?

$Q_{AB} = \Delta U_{AB} + \cancel{W_{AB}}^{\to 0} = \boxed{-723\ \text{J}}$ Heat Flows OUT

$Q_{BC} = \Delta U_{BC} + W_{BC} = 723\text{J} + 486\text{J} = \boxed{1209\ \text{J}}$ Heat Flows in

$Q_{CA} = \cancel{\Delta U_{CA}}^{\to 0} + W_{CA} = \boxed{-586\ \text{J}}$ Heat Flows out

(e) [4 points] Is this a heat engine or a refrigerator? What is its efficiency or coefficient of performance? Explain your answer.

[It is a refrigerator because negative work is done by the gas.]

$CP = \frac{|Q_L|}{|W_{NET}|}$ Q_L is Q at low temp. For a refridgerator, this is the heat taken IN to the gas FROM the cold reservoir. So $CP = \frac{1209\text{J}}{99.8\text{J}} = \boxed{12.1}$

(f) [2 points] What is the entropy change for the whole world in one complete cycle? Why?

$\boxed{\Delta S_{world} = 0}$ for a reversible process.

We know this process is reversible because we were able to draw it on a P-V diagram (i.e. it was well defined in P, V, T throughout the cycle)

4. [10 points] At a crime scene, the forensic investigator notes that the 8.2-g lead bullet that was stopped in a door-frame melted completely on impact. Assume the bullet was fired at room temperature (20°C).

The heat of fusion for lead is 5.9 kcal/kg and its melting point is 327°C.

The specific heat of lead is 0.031 cal/g*°C or 130 J/kg °C

(a) [3 points] What is the heat required to get to the melting point?

Heat $= 8.2g \times (327 - 20)°C \times 0.031 cal/g{*}$°C $=$ 78 cal

(b) [3 points] What is the heat required for melting the bullet?

Heat melting $= 8.2g \times 5.9 \times 10^3 \times 10^{-3} =$ 48.4 cal

(c) [2 points] What is the energy in joules required to heat and melt the bullet?

Energy $= 4.18 \times (78 + 48.4)$ j $=$ 528 joules

(d) [2 points] What does the investigator calculate was the *minimum* muzzle velocity of the gun?

KE $= \frac{1}{2}mv^2 =$ 528 joules.

$v = \sqrt{2 \times 528/8.2 \times 10^{-3}} = 359m/s$

5. [15 points] You plan to have iced tea for your afternoon party. You have an insulated container with 3 kg of tea (essentially water) at 20°C, to which you add ice at -10°C.

(a) [5 points] How much ice at -10°C do you need to add to the 20°C tea in order to have a resulting mixture of tea (water) and 0.2 kg of ice in equilibrium? What is the equilibrium temperature?

The equilibrium temperature will be 0° since we have ice and water together. The ice will warm and all but 0.2 kg will melt. (We will assume the specific heat capacity of ice to be the same as water; in reality it is about half.)

The heat lost by the water is

$$Q_w = m_w c_w \Delta T_w = 3000\text{g} \times 1\text{cal/g}^\circ\text{C} \times -20^\circ\text{C} = -6 \times 10^4\text{cal}$$

The heat gained by the ice is

$$\begin{aligned} Q_{ice} &= m_{ice} c_{ice} \Delta T_{ice} + (m_{ice} - 200\text{g}) L_f \\ &= m_{ice} \times 1\text{cal/g}^\circ\text{C} \times 10^\circ\text{C} + (m_{ice} - 200\text{g}) \times 80\text{cal/g} \\ &= (10 + 80) m_{ice}\text{cal/g} - 1.6 \times 10^4\text{cal} \end{aligned}$$

The two heats have to sum to zero so we have $(10+80)m_{ice}\text{cal/g} - 1.6 \times 10^4\text{cal} = 6 \times 10^4\text{cal}$ or $90 m_{ice} = 7.6 \times 10^4\text{g}$ yielding $m_{ice} = 844\text{g} = 0.844\text{kg}$

(b) [5 points] Suppose the container of the iced tea is left in the state equilibrium describe in part (a) and the temperature outside the container is fixed at 20°C. Suppose further that the container has a total surface area of 2500 cm^2, a thickness of 1 cm and is made of material with thermal conductivity 0.025 W/m°C. While there is still ice left, find the rate at which heat energy is entering the container and how long it takes the ice to melt.

We use Newton's heat conductivity equation

$$\frac{dQ}{dt} = -\kappa A \frac{\Delta T}{l} = 0.025\text{W/m}^\circ\text{C} \times 2500\text{cm}^2 \times \frac{20^\circ C}{1\text{cm}} = 0.025 \times 25 \times 20\text{W} = 12.5\text{W} = 3\text{cal/s}$$

At the end of part (a), we have 0.2 kg of ice left in our mix. The heat needed to melt that ice is $Q = m_{ice} L_f = 200\text{g} \times 80\text{cal/g} = 16000\text{cal}$. At three calories per second that takes a time $t = 16000/3\text{s} = 5350\text{s} = 89.2\text{min} = 1.49\text{h}$.

(c) [5 points] What was the entropy change in part (b) and in part (a)?

The entropy change in part (b) is very easy to calculate. For the ice water:

$$\Delta S = \Delta Q / T = 16000\text{cal}/273\text{K} = 58.6\ \text{cal/K}$$

and for the surroundings

$$\Delta S = \Delta Q/T = -16000\text{cal}/293\text{K} = -54.6\,\text{cal/K}$$

Then the net result is $\Delta S = 4.0\,\text{cal/K}$.

The entropy change in part (a) is much more complicated to calculate. In general, $dS = dQ/T$ or $\Delta S = \int dQ/T$.

For the water

$$\Delta S = \int_{293\text{K}}^{273\text{K}} m_w c_w \frac{dT}{T} = m_w c_w \ln(273/293) = 3000\text{g} \times 1\text{cal/gK}\ \ln(273/293) = -212\,\text{cal/K}$$

For the ice

$$\begin{aligned} \Delta S &= m_{ice} c_{ice} \ln(T_f/T_{ice}) + (m_{ice} - 200\text{g}) L_f/T_f \\ &= 844 \times 1\text{cal/K} \ln(273/263) + (844 - 200) \times 80\text{cal}/273\text{K} = 31.5 + 189\ \text{cal/K} \\ &= 220\,\text{cal/K} \end{aligned}$$

The total is then $\Delta S = 8\,\text{cal/K}$.

Note in both cases that $\Delta S > 0$ which is absolutely required since we are performing an irreversible process: letting heat travel from a hot object to a cold one.

6. [15 points] After learning about heat engines and pumps, your class mate decides to start a company to develop, build, sell, and install systems that use a heat pump to cool or heat houses and buildings that uses a heat engine to produce the power to run the heat pump. You classmate claims that such a system will save the user a lot of money compared to conventional systems or even a heat pump.

Consider a typical California house where the temperature difference between inside and outside is rarely more than 10°C. Because of windows, doors, cracks and other leaks, at a 10°C temperature difference heat leaks into or out of the house at the rate of about 2500 watts.

(a) [2 points] Natural gas used for heating costs about \$ 0.75 per therm. There are 2.930 KW-hr per therm. What is the cost per hour and per month (30 days) to heat that house with natural gas? A typical gas heater has 50% efficiency as a lot of the heat goes out the flue.

The heat rate needed from the natural gas is

$$\frac{dQ}{dt} = 2500\,W\left(\frac{1}{50\%}\right) = 5{,}000\,W$$

Convert this heat rate to a money rate using the given rate for natural gas:

$$\frac{d\$}{dt} = 5{,}000\,W\left(\frac{1\text{ therm}}{2{,}930\text{ W-Hr}}\right)\left(\frac{\$.75}{1\text{ therm}}\right) = \boxed{\$1.28/\text{hr}} \text{ or } \boxed{\$921.60/\text{month}}$$

(b) [2 points] What does it cost in dollars per hour and per month (30 days) to heat that house with electricity? Assume electricity costs \$ 0.1/KW-hr.

All the energy of electricity can be converted into heat, so electricity is 100% efficient. Thus, the cost rate is just

$$\frac{d\$}{dt} = 2500\,W\left(\frac{\$.1}{1000\text{ W-Hr}}\right) = \boxed{\$.25/\text{Hr}} \text{ or } \boxed{\$180/\text{month}}$$

(c) [4 points] What would it cost with an electrically powered heat pump to heat or cool that house with the 10° temperature difference? With a heat pump that was at 50% of maximum coefficient of performance?

The maximum coefficient of performance is the Carnot cycle's coefficient of performance,

$$CP_{max} = \frac{|Q_{out}|}{|W_{net}|} = \frac{|Q_H|}{|W_{net}|} = \frac{T_H}{T_H - T_L}$$

Assume the inside of the house is at 22°C ≃ 295 K.

$$CP_{max} = \frac{295\,K}{10\,K} = 29.5$$

Thus, this heat pump has a coefficient of performance of

$$CP = .5\,CP_{max} = .5(29.5) = 14.75$$

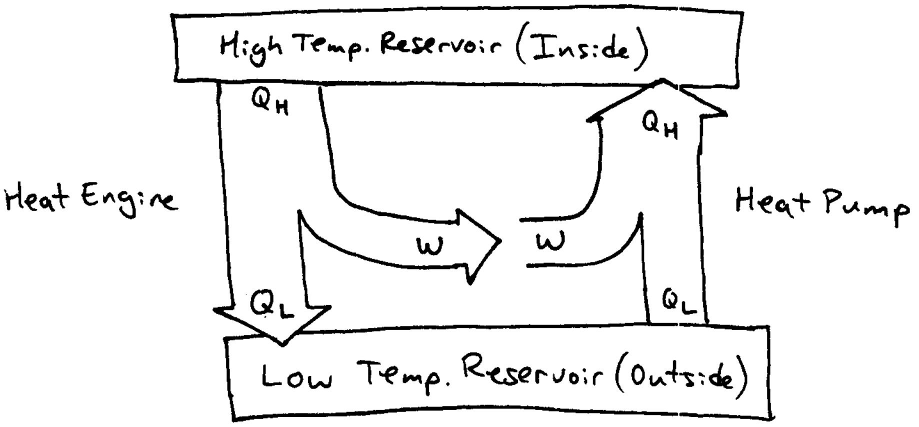
CP tells us the ratio between the heat supplied to the high temperature reservoir (the inside) and the work we must do. So, to supply heat at a rate of 2500 W, we must do work at a rate of

$$\frac{dW}{dt} = 2500W/14.75 = 169.5\,W$$

To do this work with electricity, the cost rate would be

$$\frac{d\$}{dt} = 169.5\,W\left(\frac{\$.1}{1000W\text{-}Hr}\right) = \boxed{\$.01695/hr} \text{ or } \boxed{\$12.20/month}$$

(d)



(e) If the heat engine and heat pump worked at the same efficiency, they would just cancel each other out. That is, if the work done by the heat engine were the only thing powering the heat pump, Q_H and Q_L would be the same for each machine, so there would be no net heat flow. To get a net heat flow into the high temp reservoir, Q_H must be higher for the pump than for the engine. Either the efficiency must be higher for the pump than the engine (this is not the case), or we have to add work.

To calculate the rate at which we must add work (in the form of electricity), we calculate our desired net heat flow into the high temp reservoir, and arrive at the difference in rate of work for the two machines.
we want to add 2500 W to the high temp reservoir, so

$$\frac{dQ_{H\text{-net}}}{dt} = \frac{dQ_{H\text{-pump}}}{dt} - \frac{dQ_{H\text{-engine}}}{dt} = 2500\,W$$

We find the ratio between Q_H and W for the two machines from their efficiencies.

For the engine:

$$e = \frac{W}{Q_H} = .9\, e_{carnot} = .9\,\frac{T_H - T_L}{T_H} = .9\,\frac{10K}{295K} = .0305$$

$$\dot{W}_{engine} = .0305\,\dot{Q}_{H\text{-engine}}$$

For the pump:

$$CP = \frac{Q_H}{W} = .8\,CP_{carnot} = .8\,\frac{T_H}{T_H - T_L} = .8\left(\frac{295K}{10K}\right) = 23.6$$

$$\dot{W}_{pump} = \frac{1}{23.6}\,Q_{H\text{-pump}} = .0424\,Q_{H\text{-pump}}$$

The electrical power needed is

$$P_{elec} = \dot{W}_{pump} - \dot{W}_{engine}$$

$$= .0424\,\dot{Q}_{H\text{-pump}} - .0305\,\dot{Q}_{H\text{-engine}}$$

Combining this equation with our first equation and eliminating $\dot{Q}_{H\text{-pump}}$, we get

$$\underline{P_{elec} = 105.9\,W + .0119\,Q_{H\text{-engine}}}$$

In other words, any heat pulled from the hot temperature reservoir by the heat ~~pump~~ engine will increase the amount of electricity we must provide, so the best bet is to turn the engine off. If we do this, the cost is

$$\frac{d\$}{dt} = 105.9\,W\left(\frac{\$.1}{KW\cdot hr}\right) = \boxed{\$.01059/hr} \text{ or } \boxed{\$7.63/month}$$

(f) [1 points] If the heat pump costs 2000$ per unit according to the business plan, should you invest in your fellow student's new business as a venture capitalist?

Clearly, the answer is no, since the best solution is to keep the heat engine turned off.

If the heat engine can be run in reverse as a heat pump, (with an electrical power cost of 76.27 W or $.00763/hr or $5.49/month) it might be a good heat pump business, depending on the competition.

(g) [1 points] If a heat pump system costs about 1500 dollars, a gas furnace 1000 dollars, and an electric heater system 500 dollars, which should you purchase if you need a new heating system? Hint: What would be your total cost for each of the three cases at the end of the first or second year of ownership?

	1 year	2 years
Electric :	$\$500 + 12 \times \$180 = \boxed{\$2660}$	$\$500 + 24 \times \$180 = \boxed{\$4820}$
Gas :	$\$1000 + 12 \times \$922 = \boxed{\$12,064}$	$\$1000 + 24 \times \$922 = \boxed{\$23,128}$
Heat Pump :	$\$1500 + 12 \times \$12.20 = \boxed{\$1646}$	$\$1500 + 24 \times \$12.20 = \boxed{\$1793}$

The heat pump is clearly the way to go. Note that the numbers for gas in this problem are ridiculous due to a typo in the conversion between therms and KW-Hr in the problem statement.

End of Examination