Berkeley Physics H7B Spring 2015
Dr. Winoto - Final Examination
Wednesday, May 13th, 2015

Instruction for the examination (please read carefully):
- In the front of your bluebook, next to your name, please write you SID.
- Topic covered: Electricity and Magnetism, Purcell Ch.1-9.
- You may use two 8.5”x11” sheet of notes.
- There are 5 problems, of varying levels of difficulty, do them in any order you prefer.
- You have exactly (180-10) minutes to complete the test
- **Show all your work!** Please outline and explain in details all your physical and mathematical reasonings in a clear, rational, step-by-step and logical manner.
- Cross out any parts of your written exam that you would like to discard and not considered as part of your answers.
- In this exam, *c* is always the speed of light, and *e* is always the magnitude of the charge of electron.
- Good luck!

1. (30 points): Charge plated and two electrons (see Figure 1):
Two electrons ($-e$) with equal velocities $+v$ in the +x-direction are moving side by side with a distance a apart. Midway between them is an infinite sheet of fixed positive charges with a surface charge density $+\sigma$.
(a). Please calculate how large σ has to be in order that the electrons maintain their separation a.

2. (30 points): Self-Inductance of a coaxial cable: (see Figure 2):
A kind of coaxial cable is made of a thin conducting cylinder that is surrounded by a concentric also-thin conducting cylinder. The inner cylinder has radius a, and the outer one has radius b. A current I flows up the inner cylinder, and flows back down the outer cylinder. For simplicity, assume that the cylinders have zero thickness, and that the current flows uniformly as a surface current on the surface of the cylinders.
(a). Please calculate the energy per unit length of the cable, stored in the magnetic field.
(b). Use your answer to (a) to calculate the self-inductance L of the cable per unit length.

3. (40 points): Mutual Inductance between an infinite wire and a toroid: (see Figure 3):
A straight wire carries a current I_1 up the symmetry axis (z-axis) of a square toroidal solenoid with inner radius a, outer radius b, and height h. The solenoid is tightly (or densely) wrapped with N turns of a coiled wire that carries current I_2. Using Ampere’s law, you could calculate the B_1-field of the straight wire, and in class, we calculated the B_2-field outside of the toroid to be zero, and inside it given by the following:

$$\overrightarrow{B_2}(r,\phi,z) = \frac{\mu_o}{2\pi}\frac{N I_2}{r}\hat{\phi}$$, where ϕ is the azimuthal angle w.r.t to the x-axis.

(a). Please calculate the mutual inductance M_{21} of the coil *2* due to the wire *1*.
(b). Please explicitly calculate the mutual inductance M_{12} of the wire *1* due to the coil *2*, and show that it is equal to M_{21}. Hint: one may consider the return of current I_1 is a semi-circle with an infinite radius.

4. (50 points): **Charge** oscillation in a circular parallel plate capacitor: (see Figure 4):
Consider a circular parallel plate capacitor with radius a and separation s, ($s << a$, so that we can safely ignore any fringe effect). The top plate has a time-dependent surface charge density $+\sigma(t)$, and the

bottom one $-\sigma(t)$, where $\sigma(t) = \sigma_o \sin(\omega t)$, where ω is the angular frequency and t is time. Let r be the radial distance from the axis of symmetry (the z-axis). FYI: $c\,\frac{2\pi}{\omega} >> a$, meaning that $\sigma(t)$ is varying slowly enough, but you do NOT need this information to do the problem.

(a). Calculate the magnetic field $\vec{B}(r,t)$ anywhere inside the capacitor as a function of r and t.

(b). Calculate the correction to the E-field, $\vec{E}_{\text{corr}}(r,t)$, anywhere inside the capacitor as a function of r and t. And use your result to calculate the newly modified total E-field $\vec{E}_{\text{tot}}(r,t)$ inside the capacitor as a function of r and t.

(c). Please calculate the next order correction to the $\vec{B}(r,t)$ anywhere inside the capacitor as a function of r and t.

5. (50 points): Current oscillation in a coaxial cable: (see Figure 5):

Consider the coaxial cable discussed in no. 2. A time-dependent current $I(t) = I_o \sin(\omega t)$ flows up the inner cylinder, and flows back down the outer cylinder, where ω is the angular frequency and t is time. Also here FYI: $c\,\frac{2\pi}{\omega} >>$ *b,a,length of cable*, meaning that $I(t)$ is varying slowly enough, but you do NOT need this information to do the problem. In the limit where the separation between the cylinder is very small, i.e. $b - a << a$, we can approximate the coaxial as two very large parallel plates with a very thin separation $2s$. The surface current density on the left plate is given by $\mathbb{j}(t) = \mathbb{j}_o \sin(\omega t)\,\hat{x}$, and on the right plate $\mathbb{j}(t) = -\mathbb{j}_o \sin(\omega t)\,\hat{x}$. The origin of the coordinate system is midway between the two plates.

(a). Please calculate the B-field $\vec{B}(y,t)$ in between the plates as a function of y $(-s < y < s)$ and t.

One can easily derive the vector potential of problem no.2 above, and arrives at the following approximation for the vector potential in between the parallel plates:

$$\vec{A}(\text{y}) = -\mu_o\, \mathbb{j}(t)\, y\, \hat{x}\text{ , where } -s < y < s.$$

(b). Show that this vector potential gives rise to the B-field that you calculated in part (a). Also show that the $\text{div}(\vec{A}) = 0$.

(c). Please calculate the electric field $\vec{E}(y,t)$ anywhere in between the plates as a function of y $(-s < y < s)$ and t.

(d). Calculate the correction to the B-field, $\vec{B}_{\text{corr}}(y,t)$, anywhere inside the capacitor as a function of y and t. And use your result to calculate the newly modified total B-field $\vec{B}_{\text{tot}}(y,t)$ inside the capacitor as a function of y and t.

That's all. In theory, just like in no.4, one can also calculate the next order correction to the $\vec{E}(y,t)$ anywhere inside the capacitor as a function of y and t. But, you don't have to do that in this exam.