PHYSICS 7B, Section 1 – Fall 2013
Midterm 2, C. Bordel
Monday, November 4, 2013
7pm-9pm

Make sure you show your work !

<u>Problem 1</u> - Current and Resistivity (20 pts)

a) A cable of diameter *d* carries a current *I*, and a voltage *V* is measured over a length ℓ. Determine the conductivity of the cable.

b) The temperature coefficient of resistivity for carbon is $\alpha = -5\times10^{-4}\ \mathrm{K}^{-1}$. Is carbon a metal? Explain.

c) The Tevatron at Fermilab is designed to carry a toroidal proton beam (doughnut shape), with a cross-sectional area *A* and average radius *R*. The protons, carrying an electric charge *+e* and traveling at speed *v,* create a current *I*. Calculate the number *N* of protons in the beam.

d) A wire of total length *2L* consists of two equally long pieces of wire, one made of copper (ρ_{Cu}) and the other made of aluminum (ρ_{Al}). Both wires have same diameter *d*, and a voltage *V* is applied across the length of the composite wire. What is the current *I* passing through the wire?

<u>Problem 2</u> – Electric potential (20 pts)

A non-conducting sphere of radius R_1 is surrounded by a larger but ultrathin spherical shell of radius R_2. The volume charge density of the inner sphere is $\rho_1(r)=ar$ ($a>0$).

a) Calculate the surface charge density σ_2 of the outer sphere such that its net charge is twice that of the inner sphere.
b) Calculate the electric field created at any point by this charge distribution.
c) Make a qualitative plot of the electric field as a function of the distance from the center of the spheres.
d) Set $V=0$ at infinity and calculate the electric potential created at any point.

Problem 3 – Capacitor (20 pts)

Consider a spherical capacitor made of two spherical ultrathin conducting shells connected to a battery, each carrying charge Q. The inner shell (of radius R_1) is positively charged while the outer one (of radius R_2) is negatively charged.

a) Calculate the electric field in the region separating the 2 plates.
b) Calculate the electric potential difference between the 2 plates.
c) Calculate the capacitance of this spherical capacitor.
d) If a dielectric material of dielectric constant K fills the entire space between the 2 plates, calculate the electrostatic potential energy that can be stored in this device, assuming it remains connected to a battery.

<u>Problem 4</u> – RC circuit (20 pts)

Consider the following circuit, in which the capacitor is initially uncharged.

a) Determine the time dependence of the voltage across the capacitor's plates.
b) What is the time constant τ for charging the capacitor in the circuit?
c) What is the maximum charge on the capacitor?
d) Draw the equivalent circuits for t <<τ and t>>τ.

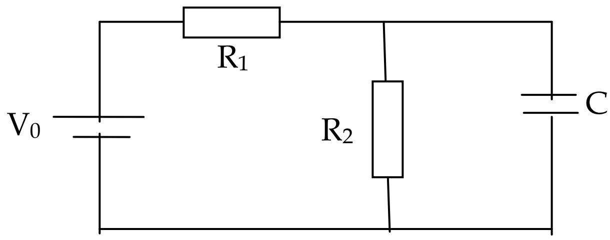

Figure 1

Problem 5 – Electric field (20 pts)

A finite size hollow cylinder of radius R and length L carries some uniform surface charge distribution σ>0.

a) What are the various methods you can effectively use in this case to calculate the electric field produced by this charge distribution on the symmetry axis (x-axis) of the cylinder? Explain.
b) Using Coulomb's law, calculate the electric field created on the symmetry axis by an infinitesimally thin ring of width $d\ell$ carrying charge dq.
c) Using part (b), calculate the electric field produced by the entire charge distribution at any point M on the symmetry axis.
d) What is the limit when $L\rightarrow\infty$? How could you get this result much more easily?

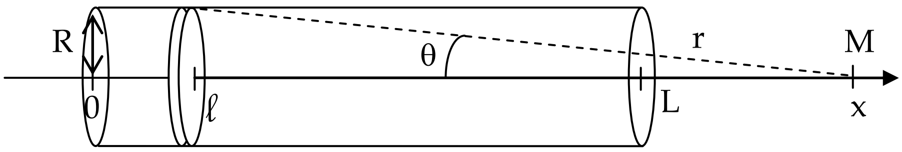

Figure 2

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$\vec{E} = \int \frac{dQ}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\rho = \frac{dQ}{dV}$$

$$\sigma = \frac{dQ}{dA}$$

$$\lambda = \frac{dQ}{dl}$$

$$\vec{p} = Q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\Delta U = Q\Delta V$$

$$V = -\int \vec{E} \cdot d\vec{l}$$

$$V = \int \frac{dQ}{4\pi\epsilon_0 r}$$

$$\vec{E} = -\vec{\nabla} V$$

$$Q = CV$$

$$C_{eq} = C_1 + C_2 \text{ (In parallel)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \text{ (In series)}$$

$$\epsilon = \kappa\epsilon_0$$

$$U = \frac{Q^2}{2C}$$

$$U = \int \frac{\epsilon_0}{2}|\vec{E}|^2 dV$$

$$I = \frac{dQ}{dt}$$

$$\Delta V = IR$$

$$R = \rho\frac{l}{A}$$

$$\rho(T) = \rho(T_0)(1 + \alpha(T - T_0))$$

$$P = IV$$

$$I = \int \vec{j} \cdot d\vec{A}$$

$$\vec{j} = nQ\vec{v_d} = \frac{\vec{E}}{\rho}$$

$$R_{eq} = R_1 + R_2 \text{ (In series)}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \text{ (In parallel)}$$

$$\sum_{\text{junction}} I = 0$$

$$\sum_{\text{loop}} V = 0$$

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial f}{\partial z}\hat{z} \\ d\vec{l} &= dr\hat{r} + r d\theta\hat{\theta} + dz\hat{z} \\ &\quad \text{(Cylindrical Coordinates)} \end{aligned}$$

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial f}{\partial \phi}\hat{\phi} \\ d\vec{l} &= dr\hat{r} + r d\theta\hat{\theta} + r\sin(\theta) d\phi\hat{\phi} \\ &\quad \text{(Spherical Coordinates)} \end{aligned}$$

$$y(t) = \frac{B}{A}(1 - e^{-At}) + y(0)e^{-At}$$
$$\text{solves } \frac{dy}{dt} = -Ay + B$$

$$y(t) = y_{max}\cos(\sqrt{A}t + \delta)$$
$$\text{solves } \frac{d^2y}{dt^2} = -Ay$$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n! 2^{2n+1}}\sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int (1 + x^2)^{-1/2} dx = \ln(x + \sqrt{1 + x^2})$$

$$\int (1 + x^2)^{-1} dx = \arctan(x)$$

$$\int (1 + x^2)^{-3/2} dx = \frac{x}{\sqrt{1 + x^2}}$$

$$\int \frac{x}{1 + x^2} dx = \frac{1}{2}\ln(1 + x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right)$$

$$\int \frac{x}{(1 + x)^{3/2}} dx = \frac{2(x + 2)}{\sqrt{1 + x}}$$

$$\frac{d\cot(x)}{dx} = -\csc^2(x)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1 + x)^\alpha \approx 1 + \alpha x + \frac{(\alpha - 1)\alpha}{2}x^2$$

$$\ln(1 + x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$