PHYSICS 7B, Lecture 2 – Spring 2014
Final exam, C. Bordel
Monday, May 12, 2014
11:30am-2:30pm

Make sure you show *all your work* and justify your answers in order to get full credit.

- **Write your name and Student ID # on your blue book.**

- **Start each problem on a new sheet.**

- **Please put the solutions in order (1-7). Note that you don't have to solve the problems in this order, you can just save at least 1 sheet (2 pages) per problem, and if you need more space then go to the end for the extra work (with a note in the original problem to guide the grader there).**

__Problem 1__ – (20 pts)

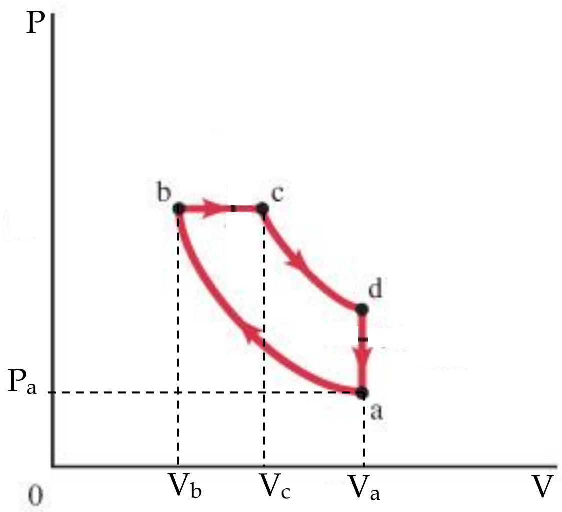
A heat engine operates in the temperature range [100-1000 K] according to the following cyclic process, where *ab* and *cd* are adiabatic processes. The engine is operated using *n* moles of CO_2, a linear molecule, considered as an ideal gas.

a) **Calculate the total heat input (Q_H) and output (Q_L), as well as the efficiency of the engine.**

The comparison of the efficiency of this engine to that of a Carnot engine requires to consider the two extreme temperatures, T_H and T_L, reached in this process.

b) **Draw on the PV diagram the isotherms associated with those two extreme temperatures.**

c) **Calculate the ratio of the efficiency of this heat engine over that of the Carnot engine operating between the same two temperatures T_H and T_L, in terms of P_a, V_a, V_b, V_c.**



__Problem 2__ – (20 pts)

An isolated spherical conductor of inner and outer radii R_1 and R_2 carries a total charge Q ($Q<0$) and contains a point charge q ($q<0$) at the center of the cavity.

a) **Calculate the electric charge and charge distribution of the conducting sphere.**

b) **Calculate the electric field and electric potential in each region of space, setting the origin of the electric potential to zero at infinity.**

c) **Make a qualitative plot of the electric field and electric potential as a function of the radial distance *r* measured from the center of the spherical conductor.**

Now the sphere is compressed, while keeping the same total charge.

d) **Calculate how much work is required to compress the sphere to an outer radius R_3 while the inner radius is held constant.**

Problem 3 – *(20 pts)*
For the design of a high voltage direct current power line, the requirement is for the power loss in the wire to be below some fraction f of the transmitted power P_{trans}.

a) **Given the length L of the line, the maximum allowable voltage drop V across the wires given a load with power consumption P_{trans}, and the resistivity ρ of the wire, calculate the minimum diameter d of the wire.**

In order to evaluate the impact of the thermal seasonal changes, let's consider a maximum temperature change ΔT.

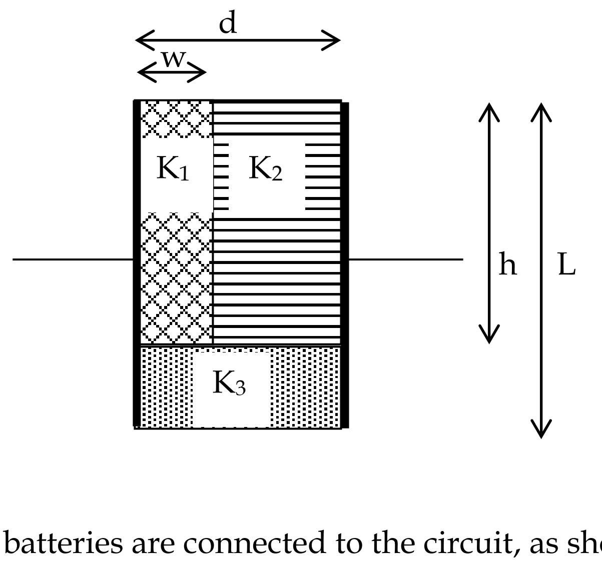
b) **Calculate the first order temperature correction to the wire cross-sectional area, length, and resistance, assuming a volumetric thermal expansion coefficient β and a temperature coefficient of resistivity α.**

Problem 4 – *(20 pts)*
A parallel-plate capacitor made of square plates (L) is filled with different dielectric materials as sketched below. You may ignore fringe effects on dielectric interfaces and assume that $d<<L$.

a) **Draw the equivalent array of capacitors, each filled with only one type of material, with their dimensions.**
b) **Calculate the capacitance C of this complex capacitor.**

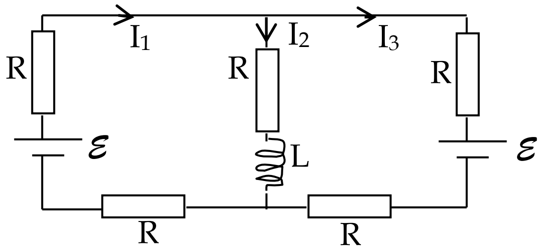
Once the capacitor is fully charged using a battery of emf $\mathcal{E}$ and a resistor of resistance R, the battery is removed so that the circuit only contains the capacitor and the resistor, and the dielectric materials are quickly pulled out of the capacitor.

c) **Sketch the voltage V_c across the capacitor's plates (still connected to the resistor) as a function of time. You may use C_k and C_0 for the capacitance with and without the dielectrics.**



Problem 5 – *(20 pts)*
At time $t=0$, the two batteries are connected to the circuit, as shown below.

a) **Calculate the current in each branch, including its direction.**
b) **Determine the limit of the voltage V_L across the inductor in the short-term and long-term regimes, and identify the equivalent component of the inductor in those two regimes.**



__Problem 6__ **– *(20 pts)***

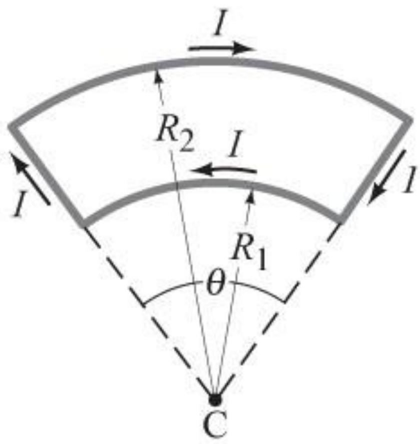
A current-carrying wire has the planar shape shown on the figure below.

a) Justify the choice of your method and determine the magnitude and direction of the magnetic field generated at point *C*.

The loop is now placed in a strong external uniform magnetic field (B_{ext}) pointing into the page. You may assume that B_{ext} is much larger than the field created by the current-carrying wire.

b) Determine the direction and magnitude of the net force acting on the loop.

c) If we allow the loop to rotate, is the loop in a stable or unstable equilibrium? *Hint: you may consider the loop of current as a magnetic dipole.*



__Problem 7__ **– *(20 pts)***

A long straight wire of radius R carries some current with a non-uniform current density $j(r)=\alpha r^2$ (α=const.), r being the radial distance measured from the symmetry axis of the wire.

a) Justify the choice of your method and calculate the magnetic field produced inside and outside the wire.

b) Draw some field lines to show qualitatively how the magnitude and direction of the magnetic field vary.

A rectangular loop of sides a and b and resistance $\mathscr{R}$ is placed at distance d ($d>R$) from the current-carrying wire, as shown below.

c) How much force does it take to move the rectangular loop at a constant speed if it is *(i)* translated along the *x*-axis at speed *v*? *(ii)* translated along the *y*-axis at speed *v*?