Physics 141A, Spring 1994

**FINAL EXAMINATION**

**Friday, May 13, 1994 12:30-3:30 pm CLOSED BOOK**

Information

1. Please cross out any work in your blue book which you do not wish to be graded. If your paper is neat, clear, and easy to read, it could affect your grade favorably.

2. Partial credit will be given for an incomplete or incorrect solution only for relevant, applicable statements that are logically presented. Random, disconnected comments will not be credited even if they happen to be correct. If you are unable to complete the answer to a question, please state clearly how far you got, and indicate how you would proceed to a solution.

3. Please indicate, where appropriate, final answers conspicuously; e.g., put answers in a box, properly labeled.

4. Several problems require that you make a sketch. A freehand sketch is entirely acceptable, but it must be clearly legible and clearly labeled!

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$e \approx 1.6 \times 10^{-19}$ C $= 4.8 \times 10^{-10}$ e.s.u.

$m_e \approx 9 \times 10^{-28}$ gm

$M_p \approx 1.7 \times 10^{-24}$gm

$\hbar \approx 10^{-27}$ erg sec $= 10^{-34}$ J sec

$k_B \approx 1.4 \times 10^{-16}$ erg K$^{-1}$ $= 1.4 \times 10^{-23}$ J K$^{-1}$

$\mu_B = e\hbar/2mc \approx 9.3 \times 10^{-21}$ erg gauss$^{-1}$ $\approx 9.3 \times 10^{-24}$ J tesla$^{-1}$

Avogadro's number $= 6 \times 10^{23}$ mole$^{-1}$

1 eV $\approx 1.6 \times 10^{-12}$ erg $\approx 1.6 \times 10^{-19}$ J

$10^4$ gauss = 1 tesla

1 statvolt = 300 volts

1. Short Problems (25 points)

(i) Two phonons with wave vectors $\vec{k}_1$ and $\vec{k}_2$ scatter.

With the aid of carefully drawn sketches explain what is meant by a normal process and an Umklapp process.

What is the order of magnitude of the minimum energy of the phonons with wave vectors $\vec{k}_1$ and $\vec{k}_2$ for an Umklapp process to occur?

Thus, explain why the probability for an Umklapp process to occur falls off as the temperature is lowered.

(ii) An atom in a one-dimensional lattice vibrates about its mean position $x = 0$ in a potential well

$$U(x) = Ax^2 - Bx^3.$$

Write down in integral form an expression for the mean displacement $\langle x \rangle$. By writing the integral(s) in a dimensionless form calculate $\langle x \rangle$, setting any numerical factors equal to zero.

Hence explain why the coefficient of expansion vanishes as the temperature $T \to 0$.

(iii) Using a sketch of the first Brillouin zone of a two-dimensional square lattice of side "a", explain qualitatively but carefully how a divalent element may be a metal, a semimetal, a semiconductor or an insulator.

(iv) Using sketches in the reduced and periodic zone schemes, explain how "open orbits" can occur when a metal is placed in a magnetic field. Why is there no build-up of charge on the edges of the metal?

(v) Using a sketch, show how the bands bend across a PN junction. The two materials are nondegenerately doped. Indicate the depletion layer, the chemical potential and the electrostatic potential that is established in thermal equilibrium.

Draw two more sketches to represent the forward and reverse bias situations, indicating in particular what happens to the electrostatic potential across the junction. With the aid of these sketches, explain qualitatively why the current increases rapidly with increasing forward bias voltage, but saturates at a limiting value for reverse bias.

2. (20 points) (a) Let R be the nearest neighbor distance (i.e. NaCℓ) bond length in NaCℓ. Write down the number of the 1st, 2nd and 3rd nearest neighbors, and their separation from a reference ion.

(b) Define the Madelung constant $\alpha$. Calculate $\alpha$ for NaCℓ summing only the first three nearest neighbors, starting with Na at the origin.

(c) Assume the repulsive energy involves first nearest neighbors only, and has the form $\lambda\exp(-R/\rho)$ for each nearest neighbor, where $\lambda$ and $\rho$ are constants. Find an expression for the total lattice energy, $U_{tot}$, for a crystal with N NaCℓ molecules, using your result in (b).

(d) From (c) find an expression involving $R_o$, the equilibrium separation of an Na and a Cℓ atom, in terms of $\alpha$, $\rho$, $\lambda$, and the electronic charge.

(e) For NaCℓ, $R_o \approx 3$Å, and $\rho \approx 0.3$Å. Find numerical values (in eV) for (i) $\lambda$, and (ii) $U_{tot}$ per NaCℓ molecule.

3. (25 points) Consider a p-type semiconductor. Assume a dielectric constant of 10 and a hole effective mass $m^*$ of 0.2 of the free mass value.

(a) By treating the motion of a hole around a negatively charged impurity atom as a hydrogen-like atom, find an analytical expression for the ionization energy. Estimate the value of the energy numerically in eV.

(b) Starting from first principles (e.g. Newton's Laws), derive an expression for the electrical conductivity of the semiconductor in terms of the carrier density, effective mass $m^*$, scattering time $\tau$ and any other parameters you deem relevant.

(c) Define the hole mobility $\mu_h$. From your answer to (b), show that $\mu_h = e\tau/m^*$. Make a rough numerical estimate of $\mu_h$ at low temperatures where $\tau = 10^{-11}$ sec.

(d) Finally, assume the semiconductor has a direct gap $E_g$. We usually assume that the minimum photon energy to excite an electron into the conduction band leaving a hole in the valence band is $E_g$. However, it is possible to use a photon of less energy if the electron and hole form a bound pair, the minimum photon energy being reduced by the pair binding energy $E_b$. The electron and hole orbit each other in the crystal, and we can estimate the binding energy by treating the electron as a charge $-|e|$ with mass $m_e$ and the hole as a charge $+|e|$ with mass $m_h$. Calculate an expression for $E_b$, and estimate its value for $\varepsilon = 10$ and $m_e^* = m_h^* = 0.2$ of the free electron mass.

4. (30 points) Consider a two-dimensional metal with a square lattice spacing of 5Å.

(i) Calculate the density of states as a function of energy (start by using periodic boundary conditions and consider a suitable two-dimensional k-space).

(ii) Calculate the Fermi energy $\varepsilon_F$ at $T = 0$ assuming N electrons per unit area. Estimate $\varepsilon_F$ in electron volts for $N = 10^{16}\ cm^{-2}$.

(iii) Now apply a magnetic field of 1 tesla ($10^4$ gauss) perpendicular to the plane of the metal. Assume the mean free path is long enough that electrons can complete many orbits before being scattered. The energy levels of the electrons become quantized and are given by $E_n = (n + \gamma)\, \hbar\omega_c$, where $\omega_c = eB/mc$ is the cyclotron frequency and $0 < \gamma < 1$.

(a) In k-space, sketch roughly the first few Landau levels, indicating how the spacing between levels depends on the magnitude of the wave vector k of the electrons in a given Landau level.

(b) Indicate how the sketch would change if the magnetic field were increased.

(c) Assuming that all the allowed k states between the $n^{th}$ and $(n + 1)^{st}$ Landau level collapse onto the $n^{th}$ level, derive an expression for the degeneracy of the $n^{th}$ level.

(d) What is approximately the quantum number n of the highest occupied Landau level?

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