Physics 141A- Spring 2004/ A. Lanzara

MIDTERM II

Tuesday April 13, 2004 8:10-9.30 am CLOSED BOOK

GOOD LUCK!

PROBLEM #1 (25 points)

Consider a line of N identical atoms of length L. There are one longitudinal mode and two transverse acoustic modes of oscillation. The speed of sound is v_s.

a) For each mode, find the density of states $D(\omega)$, using periodic boundary conditions to determine the allowed k-values (assume $\omega=v_s k$). (5pts)
b) Find the Debye frequency ω_D for each mode. (5pts)
c) Write down an integral expression for the energy U, of all three modes. (5pts)
d) From c) find the heat capacity in the classical limit $k_B T >> \hbar\omega_D$. (5pts)
e) Find an expression for the heat capacity in the quantum limit $k_B T << \hbar\omega_D$ in terms of relevant parameters and a dimensionless integral that you should not evaluate. (5pts)

PROBLEM #2 (25points)

A simple monoatomic two-dimensional square lattice is modeled by balls and springs. The lattice spacing is a, the mass of the balls is m, the spring constant of the nearest-neighbor springs (along the edges of the squares) is k_1, and the spring constant between the next-nearest neighbors (along the diagonal of the squares) is k_2. All other interactions are negligible.

Find the $\omega(k)$ relation for the longitudinal and transverse modes (15pts).

What is the frequency of the $k=(\pi/a, 0)$ mode oscillations? (5pts)

(Note: You don't need to solve completely the equations of motion. Assume small oscillations)

PROBLEM #3 (25pts)

Electrons of mass m are confined to one dimension. A weak periodic potential, described by the Fourier series $V(x)=V_0+V_1\cos(2\pi x/a) +V_2\cos(4\pi x/a)$, is applied.

(a) Under what conditions will the nearly free-electron approximation work? Assuming that the condition is satisfied, sketch the three lowest energy bands in the first Brillouin zone. Number the energy bands (starting from one at the lowest band).

(b) Calculate (to first-order) the energy gap at $k=\pi/a$ (between the first and second band) and k=0 (between the second and third band).

PROBLEM #4 (25 pts)

Assume we have a two-dimensional nearly free-electron system on a square lattice of lattice spacing a. The Fourier transform of the weak lattice potential is $V(\mathbf{G})$. We want to investigate the band structure around the $(\pi/a, \pi/a)$ point in the reciprocal lattice. The unperturbed spectrum has fourfold degeneracy at this point. Only the $\mathbf{G}= (0, 2\pi/a)$ and the $\mathbf{G}=(2\pi/a, 2\pi/a)$ components of V_G are important. Find the gap if $V_{(0,\, 2\pi/a)}=V_0$ and $V_{(2\pi/a,\, 2\pi/a)}=0$. Also, find the gap if $V_{(0,\, 2\pi/a)}=0$ and $V_{(2\pi/a,\, 2\pi/a)}=V_1$
(Hint: Draw the reciprocal lattice. You will end up with a four by four matrix!)