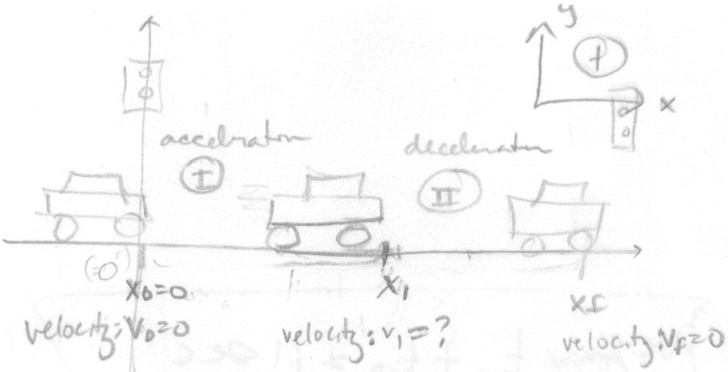


$\Delta x_{total} = ?$
(Length of block)

$\boxed{II}$
A.S
MT 1

$t_{total} = t_a + t_d = 11$ sec

$V_0 = 0$

$a_I = 4.9$ m/s

$a_{II} = -11$ m/s²

$\boxed{\text{SAMPLE APPROACH 1}}$

(I) $x_1 = \cancel{x_0}^0 + \cancel{v_0 t_a}^0 + \frac{1}{2} a_I t_a^2$

$v_1 = \cancel{v_0}^0 + a_I t_a$

$\Rightarrow x_1 = \frac{1}{2} a_I t_a^2$

$v_1 = a_I t_a$

(II) final position (for $x_0 = 0$) where we stopped after accel; how much farther we traveled during decel.

$x_f = x_1 + v_1 t_d + \frac{1}{2} a_{II} t_d^2$

$\cancel{V_f}^0 = v_1 + a_{II} t_d$

$\Rightarrow x_f = x_1 + v_1 t_d + \frac{1}{2} a_{II} t_d^2$

$\Rightarrow v_1 = -a_{II} t_d$

$t_{total} = 11 \text{ sec} = t_a + t_d$

$\Rightarrow 11 - t_a = t_d$

$V_1 = a_I t_a = -a_{II} t_d = -a_{II}(11 - t_a)$

$\Rightarrow a_I t_a = -11 a_{II} + a_{II} t_a$

$(a_I - a_{II}) t_a = -11 a_{II}$

$t_a = \dfrac{-11 a_{II}}{(a_I - a_{II})} = 7.6$ sec.

$\Rightarrow t_d = 11 - 7.6 = 3.4$ sec.

Note to all:

If you wanted to find each individual displacement and add them at the end, you must say:

$\Delta x_{accel} + \Delta x_{decel} = (x_1 - x_0) + (x_f - x_1) = \boxed{x_{tot}}$

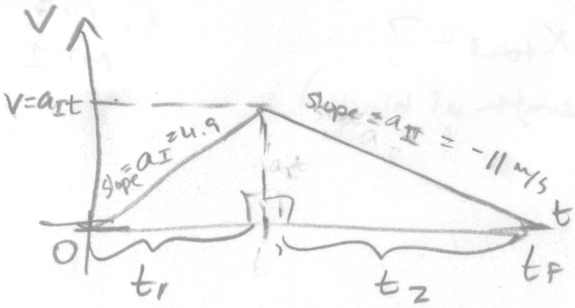
(You must do the change in each piece, otherwise, you basically add x_1 twice.)

So, $x_f = x_1 + v_1 t_d + \frac{1}{2} a_{II} t_d^2 = \frac{1}{2} a_I t_a^2 + (a_I t_a) t_d + \frac{1}{2} a_{II} t_d^2$

$= \frac{1}{2}(4.9)(7.6)^2 + (4.9 \cdot 7.6)(3.4) + \frac{1}{2}(-11)(3.4)^2$

$= \underline{205 \text{ m.}}$ //

Sample Method II



We know: $t_1 + t_2 = 11$ sec

area of v vs t ⟹ displacement!

Area: ½ base × height. Breaking into 2 triangles:

$$\Rightarrow \tfrac{1}{2} t_1 (a_I t_1) + \tfrac{1}{2} t_2 (a_{II} t_2) = L \text{ (length of block)}$$

$$= \tfrac{1}{2} a_I t_1^2 + \tfrac{1}{2}(11 - t_I)^2 a_{II} = L$$

Velocity constraint:

$$a_I t_1 + a_{II} t_2 = 0.$$ (look at the graph, or take a derivative of our area eqn)

$$\Rightarrow a_I t_1 + a_{II}(11 - t_1) = 0$$

$$(a_I - a_{II}) t_1 = -11 a_{II}.$$

$$t_1 = \frac{-11(-11)}{(4.9 - -11)} = 7.6 \text{ sec.}$$

Plug in to find $L = 205$ m. //

MT 1

SOLUTION

2)

$$h = \frac{1}{2} g t^2 \qquad \text{as } V_{oy} = 0$$

$$t = \sqrt{2h/g} \, ,$$

$$3 = v \cos 70 \, t$$

$$v_{fy} = 0 + gt$$

$$v_{fx} = v \cos 70 = 3/t \quad -(1)$$

$$v_{fy} = v \sin 70 = g t \quad -(2)$$

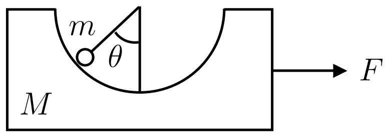
$$(2) \div (1)$$

$$\tan 70 = \frac{gt}{3/t}$$

$$t = \sqrt{\frac{3 \tan 70}{g}} = 0.917\,s$$

$$h = \frac{1}{2} g t^2 = \frac{1}{2} \times \cancel{g} \times \frac{3 \tan 70}{\cancel{g}} = \frac{3}{2} \tan \theta$$

$$= 4.12\,m.$$



Midterm 1

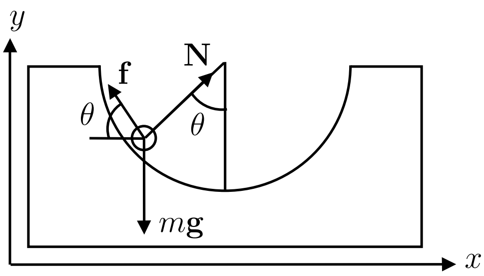
Prof. Achilles Speliotopoulos

Problem 3. A cylinder is cut out of a large block with mass M sitting on a table. A small block of negligible size and mass m is placed within the hole (see figure). The coefficient of static friction between the small block and the large block is μ_s, and you may assume that the friction between the large block and the table is negligible. What is the minimum and maximum value of F so that θ does not change?

Solution.

$$\sum \overrightarrow{F_i} = m\overrightarrow{a}$$

FBD for mass m:



axis x:

$$\sum F_x = N\sin\theta - f\cos\theta = ma \tag{1}$$

axis y:

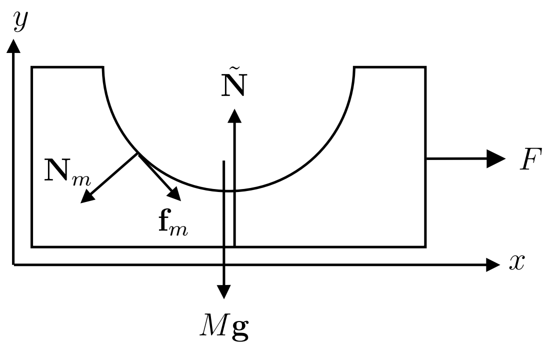
$$\sum F_y = N\cos\theta + f\sin\theta - mg = 0 \tag{2}$$

FBD for mass M:

axis x:

$$\sum F_x = -N\sin\theta + f\cos\theta + F = Ma \tag{3}$$

$$(1) + (3): \quad F = (m+M)a$$



(actually that is clear from the common sense)

In the critical case $f = \mu_s N$ and after rewriting the equations for mass m we can get:

axis x:

$$N(\sin\theta - \mu_s\cos\theta) = m\frac{F}{m+M} \tag{4}$$

axis y:

$$N(\cos\theta + \mu_s\sin\theta) = mg \tag{5}$$

$$\frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta} = \frac{F}{(M+m)g} \tag{6}$$

$$F_{min} = (M+m)g\frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta} \tag{7}$$

This case corresponds to the minimal force F_{min} since it's the force which is needed only to keep the mass m on the big block.

The case of the maximum force corresponds to the friction force in the opposite direction (all forces create the acceleration in the x direction).

If you redraw the pictures and rewrite the equations you get

$$F_{max} = (M+m)g\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} \tag{8}$$

You can see the difference between (7) and (8) only in the sign before μ_s, which is actually the change in the sign of the friction force.

4. A ball is attached by a string to a post, and is being whirled around with some speed. When the post is held still, $\theta_0 = 70°$ (see figure a). When the post is accelerated upward with acceleration a, this angle decreases and is now $\theta_{accel} = 60°$ (see figure b). What is a? You can assume that the string has negligible mass, that the speed of the ball does not change, that the ball does not bobble up and down, and that the post is still vertical to the ground.

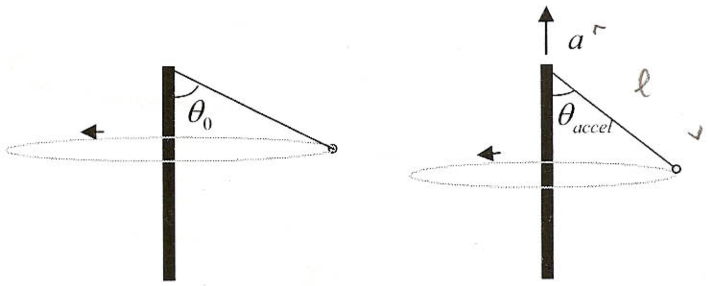
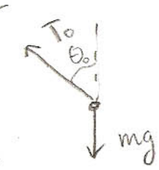


Figure a　　　　Figure b

Solution:

Initial



$$\sum F_y = T_0 \cos\theta_0 - mg = m \cdot 0 = 0.$$

$$\rightarrow T_0 = mg/\cos\theta_0$$

$$\sum F_x = T_0 \sin\theta_0 = m a_{c,0} = \frac{mv^2}{r_0} = \frac{mv^2}{\ell \sin\theta_0}$$

$$\rightarrow v^2 = T_0 \ell \sin^2\theta_0 / m$$

Accel-erated

$$\sum F_y = T_1 \cos\theta_1 - mg = ma$$

$$\sum F_x = T_1 \sin\theta_1 = m a_{c,1} = \frac{mv^2}{r_1} = \frac{mv^2}{\ell \sin\theta_1}$$

$$\rightarrow T_1 = \frac{mv^2}{\ell \sin^2\theta_1} = (T_0 \cancel{\ell} \sin^2\theta_0 / \cancel{m}) \frac{\cancel{m}}{\cancel{\ell} \sin^2\theta_1}$$

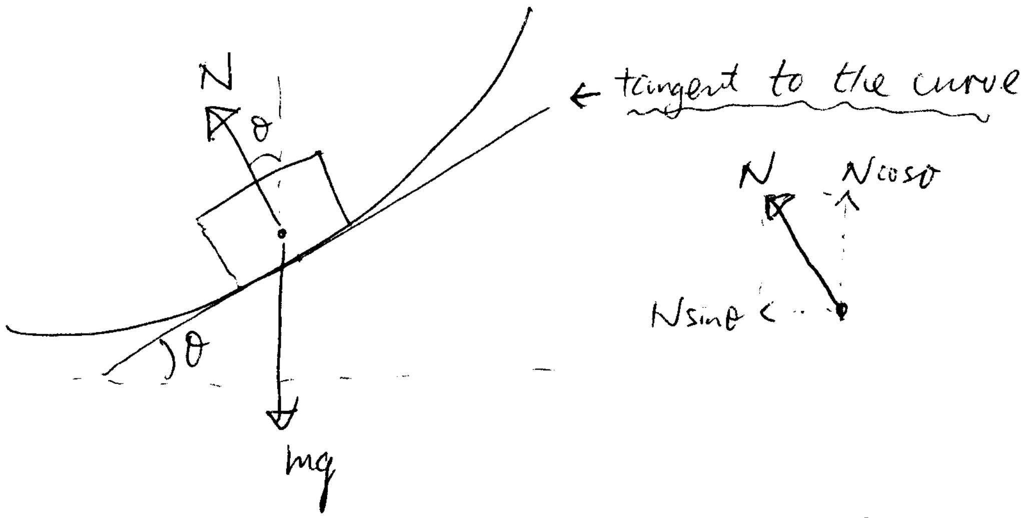
$$= T_0 \frac{\sin^2\theta_0}{\sin^2\theta_1} = \frac{mg}{\cos\theta_0} \frac{\sin^2\theta_0}{\sin^2\theta_1}$$

$$\rightarrow a = \frac{T_1}{m}\cos\theta_1 - g = g\left(\frac{\cos\theta_1}{\cos\theta_0} \frac{\sin^2\theta_0}{\sin^2\theta_1} - 1\right) = g\left(\frac{\tan\theta_0 \sin\theta_0}{\tan\theta_1 \sin\theta_1} - 1\right)$$

$$= 7.07 \ \frac{m}{s^2}$$

$(\theta_0 = 70°, \theta_1 = 60°)$

5) FBD $\boxed{\text{Solution}}$ MT 1



$$\sum F_y = mg - N\cos\theta = 0 \Rightarrow N = \frac{mg}{\cos\theta}$$

$$\sum F_x = N\sin\theta = ma \Rightarrow \frac{mg\sin\theta}{\cos\theta} = ma \Rightarrow \tan\theta = \frac{a}{g} = \frac{v^2}{gr}$$

$$\text{Slope} = \frac{dy}{dr} = \tan\theta = \frac{v^2}{gr} \Rightarrow v = \sqrt{gr\frac{dy}{dr}} \text{ ——— } (1)$$

Time to go around circle once:

$$T = \frac{2\pi r}{v} \Rightarrow v = \frac{2\pi r}{T} \text{ ——— } (2)$$

Equate (1) & (2).

$$\sqrt{gr\frac{dy}{dr}} = \frac{2\pi r}{T}$$

$$gr\frac{dy}{dr} = \left(\frac{2\pi}{T}\right)^2 r^2$$

$$\frac{dy}{dr} = \frac{4\pi^2}{T^2}\frac{r}{g}$$

$$\boxed{y = \frac{2\pi^2}{gT^2} r^2}$$, where T is constant.