Midterm 1 for MATH 53

October 7, 2014

Show your work and justify your answers.

problem	points	score
1.	12	
2.	12	
3.	12	
4.	12	
5.	12	
6.	12	
7.	12	
8.	12	
XC	4	
total	96	

1. Find the area of the region enclosed by one loop of the curve $r^2 = \sin(2\theta)$.

2. Decide if the triangle with vertices

$$P(0,-3,-4), Q(1,-5,-1), R(5,-6,-3)$$

is right-angled

(a) using angles between vectors

(b) using distances and the Pythagorean theorem.

3. Find an equation for the plane that passes through the point $(-2,4,-3)$ and is perpendicular to the planes $-x+3u-5z=42$ and $u-2z=-5$.

4. Let $\mathbf{r}(t) = \langle \sin t, 2\cos t \rangle$.

   (a) Sketch the plane curve with the given vector equation.

   (c) Sketch the position vector $\mathbf{r}(t)$ and the tangent vector $\mathbf{r}'(t)$ for the value $t = \pi/4$ (use the same graph as for (a) .

5. Find the limit, if it exists, or show that the limit does not exist.

   (a)

(b)

$$\lim_{(x,y)\to(1,0)} \frac{xy - y}{\sqrt{(x-1)^2 + y^2}}.$$

6. Use the Chain Rule to find dw/dt. Express your answer solely in terms of the variable t.

7. Find the equations of (a) the tangent plane and (b) the normal line to the given surface at the specified point.

$$x^2 + y^2 + z^2 = 3xyz, \quad (1, 1, 1).$$

8. Find the extreme values of f on the region described by the inequality.

$$f(x, y) = 2x^2 + 3y^2 - 4x - 5, \quad x^2 + y^2 \leq 16.$$

9. (Extra Credit 4 pts.)

If $\mathbf{r}(t)$ is a 3-dimensional vector-valued function having all derivatives existing, and

$$\mathbf{u}(t) = \mathbf{r}(t) \cdot [\mathbf{r}'(t) \times \mathbf{r}''(t)],$$

show that

$$\mathbf{u}'(t) = \mathbf{r}(t) \cdot [\mathbf{r}'(t) \times \mathbf{r}'''(t)].$$