1(a) Let $f(x) = e^{-|x|}$ for $|x| \le \pi$. Compute the complex Fourier coefficients $\hat{f}(k)$ relative to the interval $[-\pi, \pi]$.

(b) Evaluate

$$\sum_{k=0}^{\infty} \frac{1}{1+(2k+1)^2} = \frac{\pi}{4} \tanh\left(\frac{\pi}{2}\right).$$

(c) State a convergence theorem which justifies (b) and verify its hypotheses.

2. Let

$$f_1(x) = 1$$

$$f_2(x) = \text{sgn}(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

for $|x| \le 1$.

(a) Construct an orthonormal basis $\{\varphi_1, \varphi_2\}$ for the subspace

$$V = \text{span}\{f_1, f_2\} \subset L^2(-1,1).$$

(b) Compute the orthogonal projection of

$$f(x) = e^x$$

onto V.