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**CBE 141: Chemical Engineering Thermodynamics, Spring 2015, UC Berkeley**

**Midterm 2  
1 April 2015**

Time: 50 minutes, closed-book and closed-notes, equation sheet provided

*Please show all work and clearly mark your answers.*

*Write your name on any additional pages of scratch work.*

*Write your name on the back of the exam as well so that it is displayed when exam is folded closed.*

Point Totals:

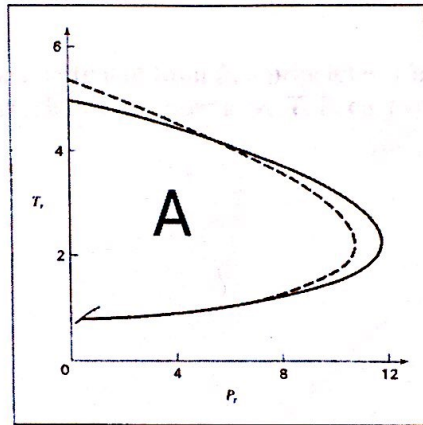
Problem 1	30	40
Problem 2	40	60
Problem 3	72	75
Total	142	175

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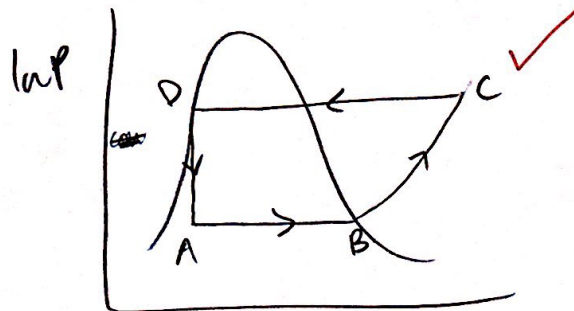
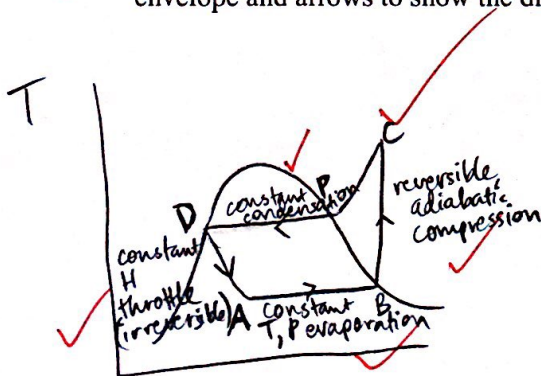
**Problem 1** (40 points)

Indicate whether the following statements (1-5) are True or False:

- 1)  F An air-standard Otto engine is operated between two constant pressures.
- 2)  T The efficiency of a Rankine cycle depends on the choice of the working fluid.
- 3)  F In a Rankine refrigeration cycle, the lower the evaporator temperature, the higher the Coefficient of Performance.
- 4)  F For a gas at any point during a reversible process,  $H^R, S^R, V^R = 0$
- 5)  T In the region A shown below, the Joule-Thomson Coefficient is positive.



- 6)  Draw a TS and a PH diagram of a Rankine refrigeration (a.k.a. vapor-compression) cycle and identify the operations involved. Use letters to indicate nodes, include the phase envelope and arrows to show the direction of the processes.



same operations for each pair of letters as T. (e.g. from A → B, B → C, etc)

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- 7) Derive the Joule-Thomson coefficient from first principles. Calculate the value of this coefficient for an ideal gas (choose representative T, P, etc to do so, if necessary).

$$\mu = \left( \frac{\partial T}{\partial P} \right)_H$$

X

$$\mu = \left( \frac{\partial T}{\partial P} \right)_H$$

10

$$PV = RT, T = \frac{PV}{R}$$

For ideal gas:

$$\mu = \frac{V}{R}$$

X  $\mu_{JT} = 0$  for IG

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Problem 2 (60 points)

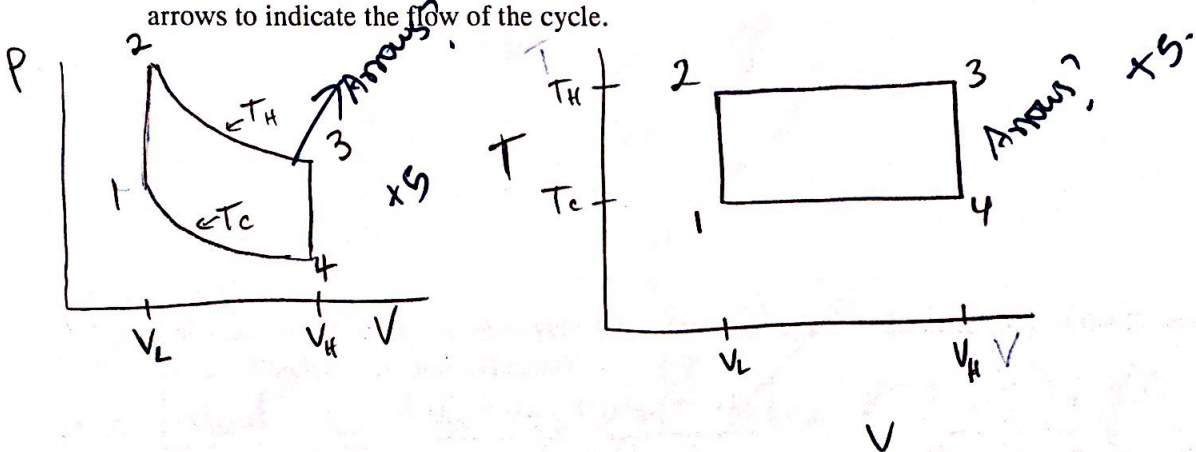
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Consider a new cycle you have not seen before—the Stirling cycle! The four steps are:

- Step 1-2: Isochoric heating
- Step 2-3: Isothermal expansion
- Step 3-4: Isochoric cooling
- Step 4-1: Isothermal compression

Assume an ideal gas serves as the working fluid and that  $C_p = (5/2)R$  and  $C_v = (3/2)R$ .

(a) Draw this cycle on a T-V and P-V diagram. Label the steps with numbers and show arrows to indicate the flow of the cycle.



(b) Calculate the heat and work for each of the steps in terms of measurable properties (e.g. T, P, V)

1-2:  $Q = C_v \Delta T = \boxed{\frac{3}{2} R (T_H - T_c) = Q}$   $W = 0$  ✓  $+2.5$  *E balance??*

2-3:  $Q = -W = \int P dV = \int \frac{RT}{V} dV = \boxed{RT_H \ln\left(\frac{V_H}{V_L}\right) = Q}$   $W = -RT_H \ln\left(\frac{V_H}{V_L}\right)$  ✓  $+2.5$

3-4:  $Q = C_v \Delta T = \boxed{\frac{3}{2} R (T_c - T_H) = Q}$   $W = 0$  ✓  $+2.5$

4-1:  $Q = -W = \int P dV = \int \frac{RT}{V} dV = \boxed{RT_c \ln\left(\frac{V_L}{V_H}\right) = Q}$   $W = -RT_c \ln\left(\frac{V_L}{V_H}\right)$  ✓  $+2.5$

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(c) Calculate the efficiency of this cycle. How does it compare to the efficiency of the Carnot cycle? Which one is more efficient?

$$\eta = \frac{|W_{\text{net}}|}{Q_{12} + Q_{23}} \times 5 = \frac{|W_{12} + W_{23} + W_{34} + W_{41}|}{Q_{12} + Q_{23}} \times 5 = \frac{\cancel{R}T_H \ln\left(\frac{V_H}{V_L}\right) + \cancel{R}T_C \ln\left(\frac{V_L}{V_H}\right)}{\frac{3}{2}\cancel{R}(T_H - T_C) + \cancel{R}T_H \ln\left(\frac{V_H}{V_L}\right)} \times 5$$

$$\eta = \frac{T_H \ln\left(\frac{V_H}{V_L}\right) - T_C \ln\left(\frac{V_H}{V_L}\right)}{\frac{3}{2}(T_H - T_C) + T_H \ln\left(\frac{V_H}{V_L}\right)} \times 5 = \frac{\ln\left(\frac{V_H}{V_L}\right)(T_H - T_C)}{\frac{3}{2}(T_H - T_C) + T_H \ln\left(\frac{V_H}{V_L}\right)} \times 5$$

$$\text{Carnot efficiency} = 1 - \frac{T_C}{T_H} \times 5$$

Carnot is more efficient ✓  
× 5.

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**Problem 3 (75 points)**

A gas follows the equation of state:

$$P = \frac{RT}{V-b} \quad v = b + \frac{RT}{P}$$

with  $b = 10^{-4} \frac{\text{m}^3}{\text{mol}}$ . Its ideal-gas heat capacity at constant pressure is  $C_p = 36.3 + \frac{T}{100} \frac{\text{J}}{\text{mol K}}$ .

The gas is passed through a pump and heat exchanger so that it is heated and pressurized from  $(T_1, P_1) = (300 \text{ K}, 1 \text{ bar})$  to  $(T_2, P_2) = (400 \text{ K}, 10 \text{ bar})$ . Calculate the differences in molar properties  $\Delta H$ ,  $\Delta S$ , and  $\Delta U$  for this change of state.

Recall that  $R = 8.314 \frac{\text{J}}{\text{mol K}}$ .

You must show all work and derivations of equations in order to receive full credit. Please indicate your final answers clearly.

Hint: A convenient place to start is by writing total derivatives for U, H, and S in terms of T and P.

$$dH = \left(\frac{\partial H}{\partial T}\right)_P dT + \left(\frac{\partial H}{\partial P}\right)_T dP \quad \checkmark$$

$$dH = C_p dT + \left[ T \left(\frac{\partial S}{\partial P}\right)_T + V \right] dP \quad \checkmark$$

$$\text{or } dH = C_p dT + \left[ V - T \left(\frac{\partial V}{\partial T}\right)_P \right] dP \quad \checkmark$$

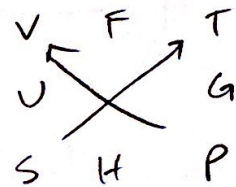
$$dH = C_p dT + \left[ b + \frac{RT}{P} - \frac{RT}{P} \right] dP$$

$$dH = C_p dT + b dP$$

$$\Delta H = \int_{300}^{400} C_p dT + b \int_1^{10} dP$$

$$\Delta H = \int_{300}^{400} \left[ 36.3 + \frac{T}{100} \right] dT + 10^{-4} \int_{1 \times 10^5}^{10 \times 10^5} dP$$

$$= \boxed{4070 \text{ J/mol} = \Delta H} \quad \checkmark$$



$$dH = T dS + V dP$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$dU = T dS - P dV$$

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$dG = -S dT + V dP$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$

**CBE 141: Chemical Engineering Thermodynamics, Spring 2015**

**Midterm 2 Equation Sheet**

Equations of State

$$PV = RT$$

$$Z = \frac{PV}{RT} = 1 + \frac{B}{V} + \frac{C}{V^2} + \frac{D}{V^3}$$

$$P = \frac{RT}{V-b} - \frac{a}{V^2}$$

$$P = \frac{RT}{V-b} - \frac{aa}{V(V+b)}$$

Thermodynamic Potentials

$$A = U - TS$$

$$H = U + PV$$

$$G = H - TS$$

Properties of Derivatives for  $z(x,y)$

$$\frac{\partial^2 z}{\partial x \partial y} = \left[ \frac{\partial}{\partial x} \left( \frac{\partial z}{\partial y} \right) \right]_x = \left[ \frac{\partial}{\partial y} \left( \frac{\partial z}{\partial x} \right) \right]_y$$

$$\left( \frac{\partial z}{\partial y} \right)_x \left( \frac{\partial y}{\partial x} \right)_z \left( \frac{\partial x}{\partial z} \right)_y = -1$$

$$\left( \frac{dz}{dy} \right)_x = \frac{1}{\left( \frac{dy}{dz} \right)_x}$$

$$dz = \left( \frac{dz}{dx} \right)_y dx + \left( \frac{dz}{dy} \right)_x dy$$

Gibbs' Phase Rule

$$F = C - \pi + 2$$