Physics 7A, Section 2 (Prof. Hallatschek)
Final, Fall 2014
Berkeley, CA

Rules: *This midterm is closed book and closed notes. You are allowed two sides of one-half sheet of 8.5" x 11" of paper on which you can whatever note you wish. You are also allowed to use scientific calculators in general, but not ones which can communicate with other calculators through any means.* **Anyone who does use wireless-capable will automatically receive a zero for this midterm.** *Cell phones must be turned off during the exam, and placed in your backpacks.* ***In particular, cell-phone-based calculators cannot be used.***

Please make sure that you do the following during the midterm:

- *Write your name, discussion number, ID number on all documents you hand in.*
- *Make sure that the grader knows what s/he should grade by circling your final answer.*
- *Answer all questions that require a numerical answer to three significant figures.*

We will give partial credit on this midterm*, so if you are not altogether sure how to do a problem, or if you do not have time to complete a problem, be sure to write down as much information as you can on the problem. This includes any or all of the following: Drawing a clear diagram of the problem, telling us how you would do the problem if you had the time, telling us why you believe (in terms of physics) the answer you got to a problem is incorrect, and telling us how you would mathematically solve an equation or set of equations once the physics is given and the equations have been derived. Don't get too bogged down in the mathematics; we are looking to see how much physics you know, not how well you can solve math problems.*

If at any point in the exam you have any problems, just raise your hand, and we will see if we are able to answer it.

Name: ____________________ *Disc Sec Number:* ____________________

Signature: ____________________ *Disc Sec GSI:* ____________________

Student ID Number: ____________________

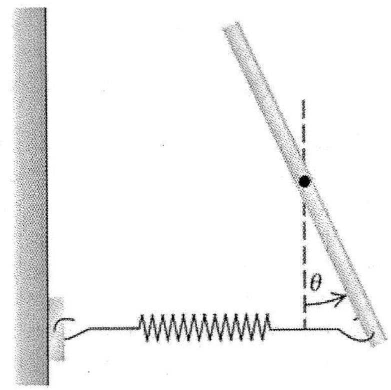
Problem	***Possible***	***Score***
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. A large beetle is standing on the middle of a circus tightrope that is stretched with tension T_s. The rope has mass per unit length μ. Wanting to shake the beetle off the rope, a tightrope walker moves her foot up and down near the end of the tightrope, generating a sinusoidal transverse wave of wavelength λ and amplitude A. Assume that the magnitude of the acceleration due to gravity is g.

What is the minimum wave amplitude A_{min} such that the beetle will become momentarily "weightless" at some point as the wave passes underneath it? Assume that the mass of the beetle is too small to have any effect on the wave propagation. Express the minimum wave amplitude in terms of the parameters given above.

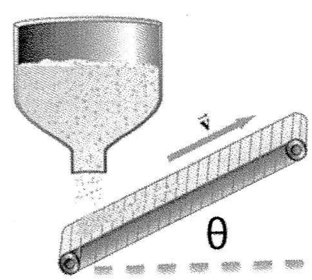
2. A slender, uniform metal rod of mass M and length l is pivoted without friction about an axis through its midpoint and perpendicular to the rod. A horizontal spring, assumed massless and with force constant k, is attached to the lower end of the rod, with the other end of the spring attached to a rigid support. The spring is relaxed when the rod is in the vertical position (θ=0).

Assume throughout that θ is small enough that the spring remains effectively horizontal and you can approximate $\sin(\theta)\approx\theta$ (and $\cos(\theta)\approx1$).



a) What is the angular frequency ω of oscillations of the rod? Express the angular frequency in terms of parameters given in the introduction.
b) Suppose the rod is cut just above the pivot point, such that the upper half of the rod is removed. What is the angular frequency ω of oscillations of the remaining system?

3. You are designing a conveyor system for a gravel yard. A hopper drops gravel of mass $\rho\,\Delta t$ in a time interval Δt onto an inclined conveyor belt (ρ is a "mass per unit time"). The conveyor belt moves at a constant speed v (see the figure).



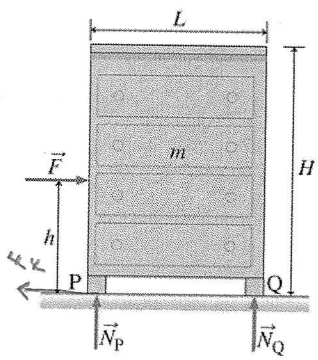
Suppose the conveyor belt is retarded by a friction force of F_{Fr}. Determine the required output power of the motor as a function of time from the moment gravel first starts falling (t=0) until the gravel begins to be dumped off the end of the conveyor belt of length L.

4. Tidal forces are gravitational forces exerted on different parts of a body by a second body. Their effects are particularly visible on the earth's surface in the form of tides. To understand the origin of tidal forces, consider the earth-moon system to consist of two spherical bodies, each with a spherical mass distribution. Let r_e be the radius of the earth, m be the mass of the moon, and G be the gravitational constant.

a) Let r denote the distance between the center of the earth and the center of the moon. What is the magnitude of the acceleration a_e of the earth due to the gravitational pull of the moon? Express your answer in terms of G, m, and r.

b) Since the gravitational force between two bodies decreases with distance, the acceleration a_{near} experienced by a unit mass located at the point on the earth's surface closest to the moon is slightly different from the acceleration a_{far} experienced by a unit mass located at the point on the earth's surface farthest from the moon. Give a general expression for the quantity $a_{near}-a_{far}$. Express your answer in terms of G, m, r, and r_e.

5. Sam is trying to move a dresser of mass m and dimensions of length L and height H by pushing it with a horizontal force $\vec{F}$ applied at a height h above the floor. The coefficient of kinetic friction between the dresser and the floor is μ_k and g is the magnitude of the acceleration due to gravity. The ground exerts upward normal forces of magnitudes N_P and N_Q at the two ends of the dresser. Note that this problem is two-dimensional.

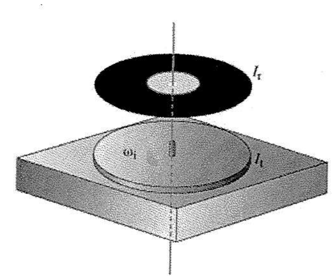


a) If the dresser is sliding with constant velocity, find F, the magnitude of the force that Sam applies. Express the force in terms of m , g, and μ_k.

b) Find the magnitude of the normal forces N_P and N_Q. Assume that the legs are separated by a distance L, as shown in the figure.
Express this normal force in terms of m, g, L, h, and μ_k.

c) Find h_{max}, the maximum height at which Sam can push the dresser without causing it to topple over. Express your answer for the maximum height in terms L and μ_k.

6. Consider a turntable to be a circular disk of moment of inertia I_t rotating at a constant angular velocity ω_i around an axis through the center and perpendicular to the plane of the disk (the disk's "primary axis of symmetry"). The axis of the disk is vertical and the disk is supported by frictionless bearings. The motor of the turntable is off, so there is no external torque being applied to the axis. Another disk (a record) is dropped onto the first such that it lands coaxially (the axes coincide). The moment of inertia of the record is I_r. The initial angular velocity of the second disk is zero.
Note that there *is* friction between the two disks.



After this "rotational collision," the disks will eventually rotate with the same angular velocity.

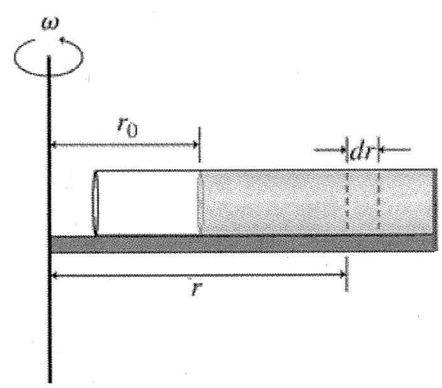
a) What is the final angular velocity, ω_f, of the two disks?
Express ω_f in terms of I_t, I_r, and ω_i.

b) Because of friction, rotational kinetic energy is not conserved while the disks' surfaces slip over each other. What is the final rotational kinetic energy, K_f, of the two spinning disks? Express the final kinetic energy in terms of I_t, I_r, and the initial kinetic energy K_i of the two-disk system. No angular velocities should appear in your answer.

c) Assume that the turntable decelerated during time Δt before reaching the final angular velocity (Δt is the time interval between the moment when the top disk is dropped and the time that the disks begin to spin at the same angular velocity). What was the average torque, $\langle \tau \rangle$, acting on the bottom disk due to friction with the record?
Express the torque in terms of I_t, ω_i, ω_f, and Δt.

d) What is the actual torque, τ, acting on the bottom disk due to friction with the record if you assume that the coefficient of kinetic friction between both disks is μ? Use your result to determine Δt. (*Hint:* Find the angular momenta generated by friction forces that act in a ring of radius r. Then, integrate over r.)

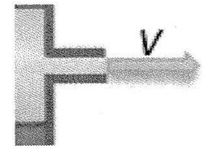
7. A test tube (cross-sectional area A) filled with liquid of uniform density ρ, as shown in the figure, is spun in a centrifuge with angular frequency ω.



The test tube lies perpendicular to the axis of rotation of the centrifuge. The pressure in the fluid at the distance r_0 from the axis of rotation is p_0. You may ignore the variation in pressure with depth; assume that it is much smaller than the variation in pressure with radius.

a) What is the pressure $p(r)$ of the fluid in the test tube at an arbitrary distance r from the axis of rotation?
Express your answer in terms of ρ, r, ω, r_0, and p_0.

b) Suppose, a small hole of cross-sectional area a is drilled at the bottom of the test tube so that liquid flows out of the test tube. At what speed v does the liquid flow out at a moment when the liquid is filled up to a distance r_0 from the axis of rotation?
Assume that the hole is so small ($a \ll A$) that the pressure as a function of r has the same form as in a) for a given filling level r_0.



c) How long will it take to empty the test tube?
Hint: You will need the integral

$$\int_a^b (b^2 - x^2)^{-1/2}\, dx = \cos^{-1}(a/b)$$

where $\cos^{-1}$ is the inverse function of the cosine and $0<a<b$ holds.