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University of California at Berkeley
Electrical Engineering and Computer Science
EE105 Midterm Examination #2
April 9, 2015
(80 minutes)

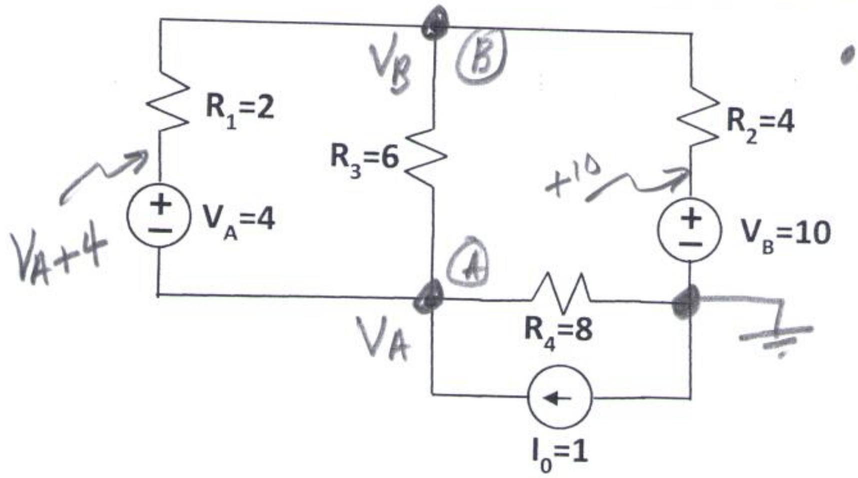
CLOSED BOOK; Two standard 8.5" x 11" sheets of notes (both sides) permitted

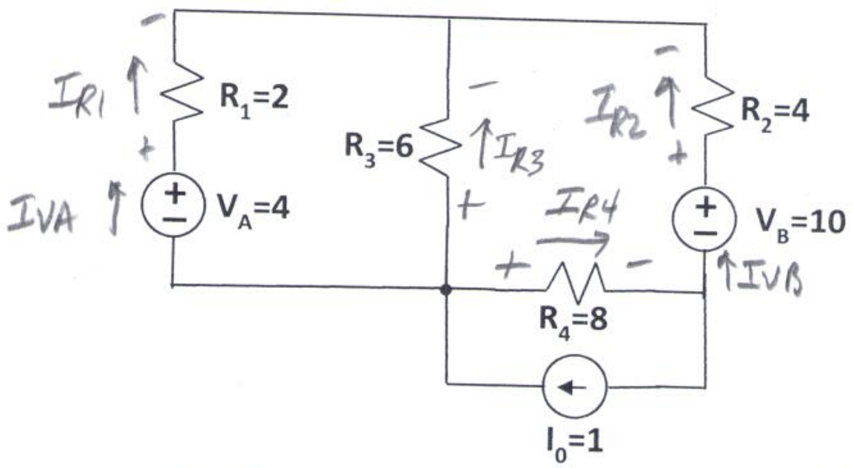
IMPORTANT NOTES

- Read each problem completely and thoroughly before beginning to work on it
- Summarize all your answers in the boxes provided on these exam sheets
- Show your work in the space provided so we can check your work and scan for partial credit
- Remember to put your name in the space above

Problem #	Points Possible	Score
1	14	14
2	18	18
3	24	24
4	16	16
Total	72	72

1. Circuit Analysis (14 points) Find the current in each element. Show each step in your solution to receive full credit. Two copies of the same schematic are shown in case you need more than one.





• Extraordinary node is one with at least 3 branches. Thus, there are three as indicated. Choose one as a ground node with the others unknown known voltages as shown. Then, account for the two voltage sources as shown.

KCL @ A:

$24 + 19V_A - 16V_B = 0$ ... (1)

KCL @ B:

$-54 - 8V_A + 11V_B = 0$ ... (2)

Solving gives: $\boxed{V_A = 7.41\,V \quad V_B = 10.30\,V}$

$I_{R1} = \dfrac{(V_A+4) - V_B}{2} = 0.55\,A$

$I_{R2} = \dfrac{10 - V_B}{4} = -0.08\,A$

$I_{R3} = \dfrac{V_A - V_B}{6} = -0.48\,A$

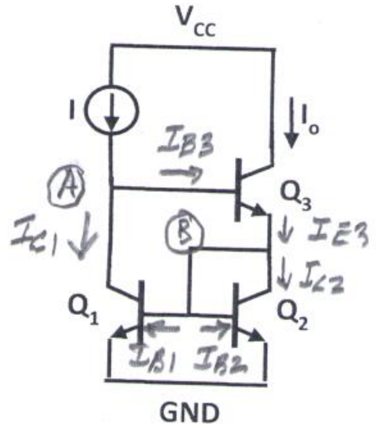
$I_{R4} = \dfrac{V_A - 0}{8} = 0.93\,A$

$I_{VA} = I_{R1} = 0.55\,A$

$I_{VB} = I_{R2} = -0.08\,A$

Element	Current (A)
I_{R1}	= 0.55 A
I_{R2}	= − 0.08 A
I_{R3}	= − 0.48 A
I_{R4}	= 0.93 A
I_{VA}	= 0.55 A
I_{VB}	= − 0.08 A
I_{I0}	= 1 A

2. Bipolar Junction Transistor DC Bias (18 points). Find expressions for the Quiescent Points for Q_1-Q_3 in terms of β and the reference current I. All transistors are identical and operate in the Forward Active Region (F.A.R.). Neglect the Early effect.



F.A.R. $\Rightarrow I_C = I_S(e^{V_{BE}/V_T} - 1)$

$V_{BE_1} = V_{BE2} \therefore I_{C1} = I_{C2}$

KCL @ Ⓐ: $I = I_{C1} + I_{B3}$ - - - - ①

KCL @ Ⓑ: $I_{E3} = I_{B1} + I_{B2} + I_{C2}$

$= \frac{I_{C1}}{\beta} + \frac{I_{C1}}{\beta} + I_{C1} = I_{C1}\left(1 + \frac{2}{\beta}\right)$ ... ②

Now, substitute ② into ①:

$$I = I_{C1} + I_{B3} = I_{C1} + \frac{I_{E3}}{\beta+1} = \left(\frac{\beta + 2 + 2/\beta}{\beta+1}\right) I_{C1}$$

$$\therefore I_{C1} = I_{C2} = \frac{(\beta+1)\,I}{(\beta + 2 + 2/\beta)} \;;\quad I_{C3} = \frac{\beta}{\beta+1} I_{E3} = \frac{(\beta+2)\,I}{(\beta+2+2/\beta)}$$

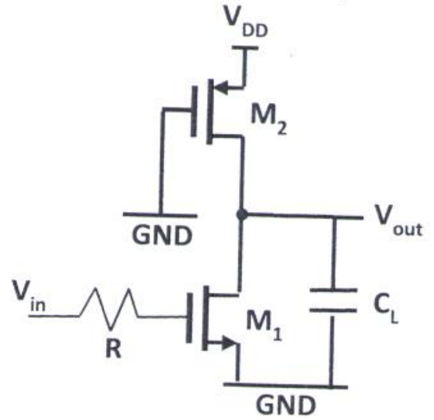
$V_{CE_1} = V_{C1} - V_{E1} = (V_{BE2} + V_{BE3}) - 0 = 2V_{BE}$

$V_{CE2} = V_{C2} - V_{E2} = V_{BE2} - 0 = V_{BE}$

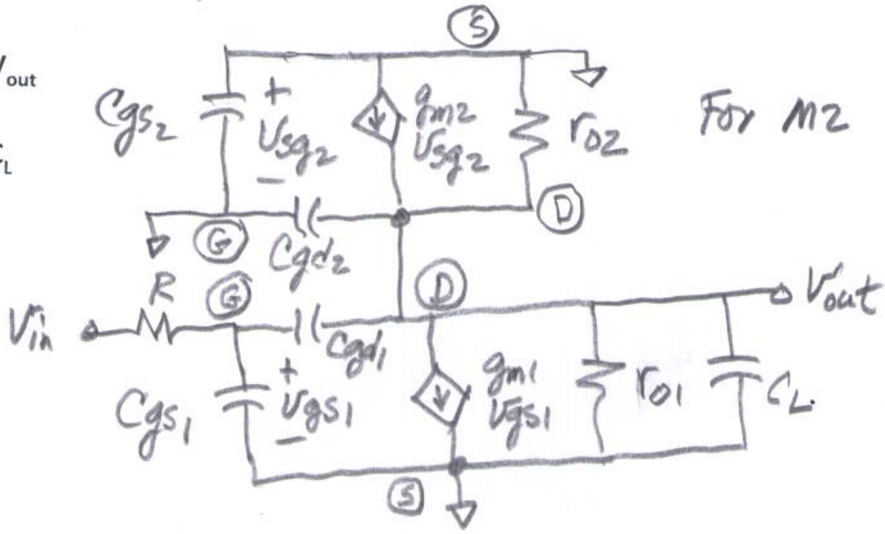
$V_{CE3} = V_{C3} - V_{E3} = V_{CC} - V_{BE}$

Parameter	Expressions
For Q_1:	$I_{C1} = \frac{(\beta+1)\,I}{(\beta+2+2/\beta)}$
	$V_{CE1} = 2V_{BE}$
For Q_2:	$I_{C2} = \frac{(\beta+1)\,I}{(\beta+2+2/\beta)}$
	$V_{CE2} = V_{BE}$
For Q_3:	$I_{C3} = \frac{(\beta+2)\,I}{(\beta+2+2/\beta)}$
	$V_{CE3} = V_{CC} - V_{BE}$

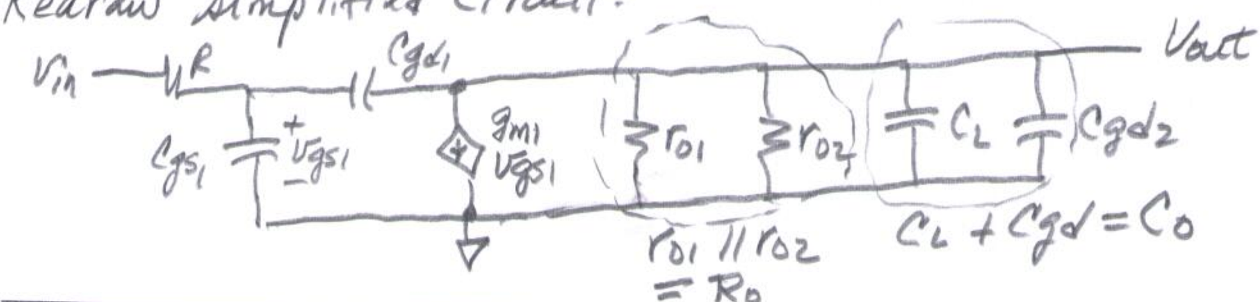
3. **Dominant Pole Approximation: [24 points].** Derive an expression for the dominant pole frequency, ω_p, of the circuit below using the open-circuit time constant (OCTC) method. Assume M_1 and M_2 operate in the saturation region. Do not use the Miller Effect. (Note: Next page is blank and available for your solution if you need more space.)



First, draw the complete small-signal model for M1 and M2



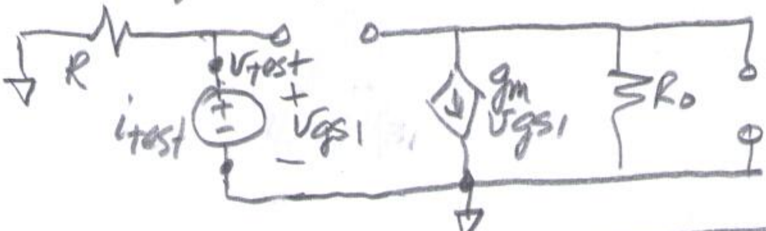
- For M2:
  - C_{gs2} is shorted — neglect
  - g_{m2} is zero because $V_{sg2} = 0$ — neglect
  - C_{gd2} from Vout to gnd; r_{o2} from Vout to gnd
- Redraw simplified circuit:



Parameter	Expression (w/ units)
ω_p	$= \left[R C_{gs1} + R_O C_O + (R_O + R + g_{m1} R_O R) C_{gd1} \right]^{-1}$ where $R_O = r_{o1} \parallel r_{o2}$ and $C_O = C_L + C_{gd2}$

3. (cont – Blank page for solution) • Now, use OCTC method

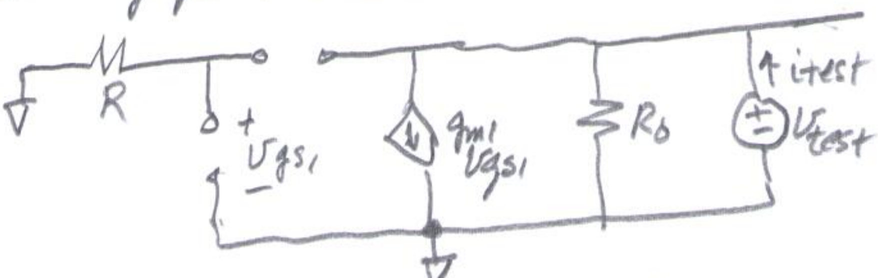
(i) Find driving point resistance for C_{gs1} with C_{gd1} and C_{gd2} open circuited:



τ_1

$\therefore \dfrac{v_{test}}{i_{test}} = R \quad \therefore \boxed{\tau_1 = R\,C_{gs1}}$

(ii) Driving point resistance for C_o with C_{gs1} and C_{gd1} open:

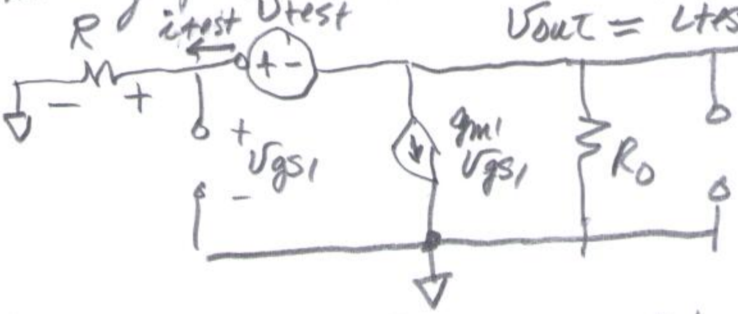


τ_2

$v_{gs1} = 0 \;\therefore\; g_{m1} v_{gs1} = 0 \;\therefore\; \dfrac{v_{test}}{i_{test}} = R_o \;\therefore\; \boxed{\tau_2 = R_o C_o}$

(iii) Driving point resistance for C_{gd1} with C_{gs1} and C_o open:

$v_{out} = i_{test} R - v_{test}$



$v_{gs1} = i_{test} R$

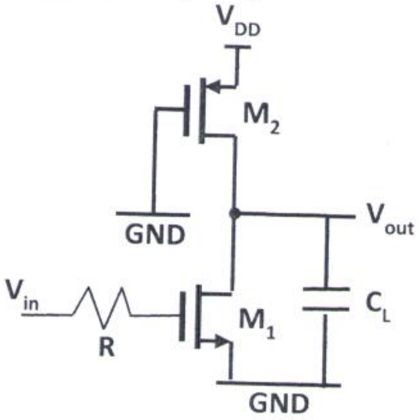
$\therefore g_{m1} v_{gs1} = g_{m1} R\, i_{test}$

• KCL at v_{out}: $i_{test} + g_{m1} R\, i_{test} + g_o i_{test} R - g_o v_{test} = 0$

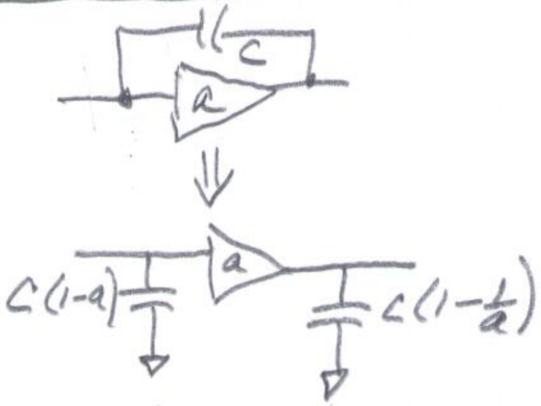
$\therefore \dfrac{v_{test}}{i_{test}} = R_o(1 + g_{m1} R + g_o R)$
$= R_o + R + g_{m1} R R_o \quad \therefore \boxed{\tau_3 = C_{gd1}(R_o + R + g_{m1} R R_o)}$

$\omega_p \simeq \dfrac{1}{\tau_1 + \tau_2 + \tau_3}$

4. Dominant Pole Approximation: [16 points]. Derive an expression for the dominant pole frequency, ω_p, of the circuit below using the Miller Effect, as appropriate. Assume M_1 and M_2 operate in the saturation region. (Note: Next page is blank and available for your solution if you need more space.)

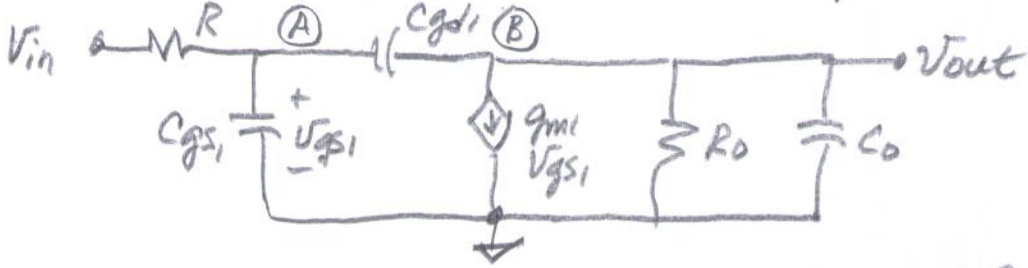


Recall Miller Effect:

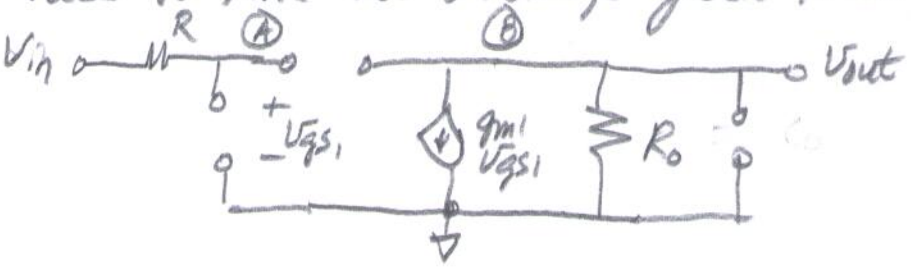


$\simeq C$ for a large.

Here is the simplified circuit from problem 3:



Note: Cgd1 is connected between Ⓐ and Ⓑ so we need to find the voltage gain from Ⓐ to Ⓑ.



KCL @ Ⓑ:

$A_v = \frac{V_{out}}{V_{in}}$

$= -g_{m1} R_o$

$= -g_{m1} (r_{o1} \| r_{o2})$

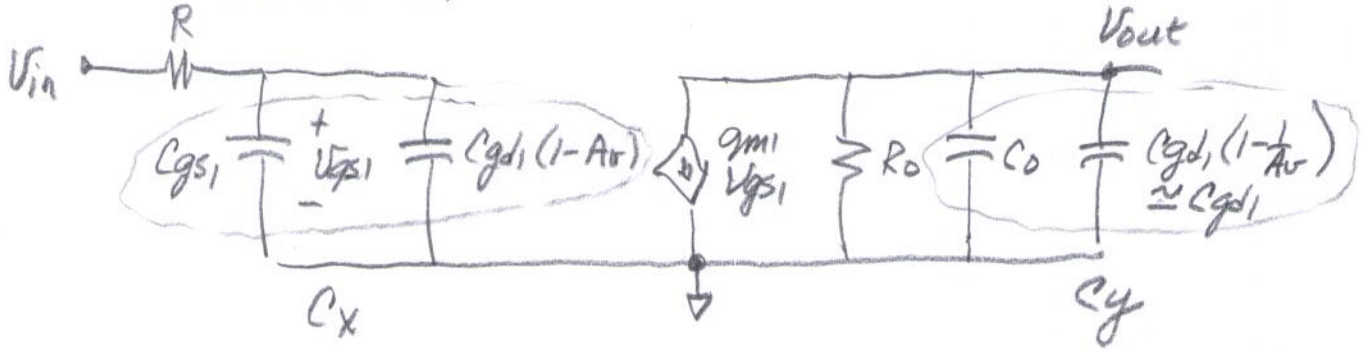
Parameter	Expression (w/ units)
ω_p	$= \left[R\left(C_{gs1} + C_{gd1}(1+g_{m1}R_o)\right) + R_o(C_o + C_{gd1})\right]^{-1}$

Factor to be like page 4 solution:

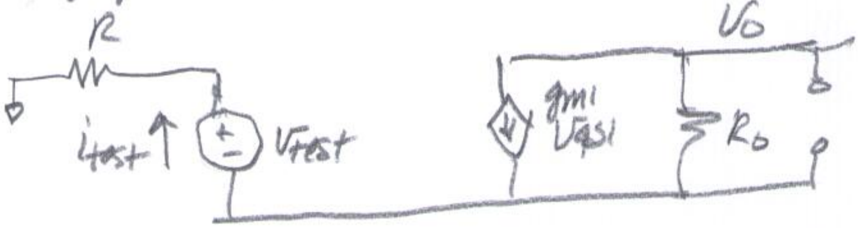
$= \left[RC_{gs1} + R_oC_o + (R_o + R + g_{m1}RR_o)\right]^{-1}$

SAME RESULT!

4. (cont – Blank page for solution)



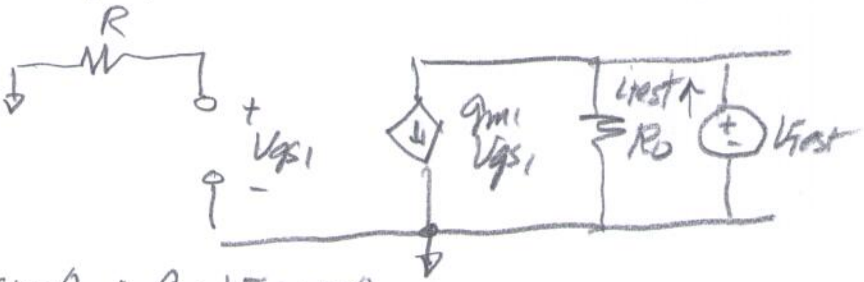
(i) Driving point resistance for C_X:



τ_1

$$\frac{v_{test}}{i_{test}} = R \;\therefore\; \tau_4 = RC_X = R\left(C_{gs1} + C_{gd1}(1+g_{m1}R_o)\right)$$

(ii) Driving point resistance for C_Y:



$v_{gs1}=0 \;\therefore\; g_{m1}v_{gs1}=0$

$$\therefore\; \frac{v_{test}}{i_{test}} = R_o \;\therefore\; \tau_5 = R_o\left(C_o + C_{gd1}\right)$$

$$\omega_p \simeq \frac{1}{\tau_4 + \tau_5}$$