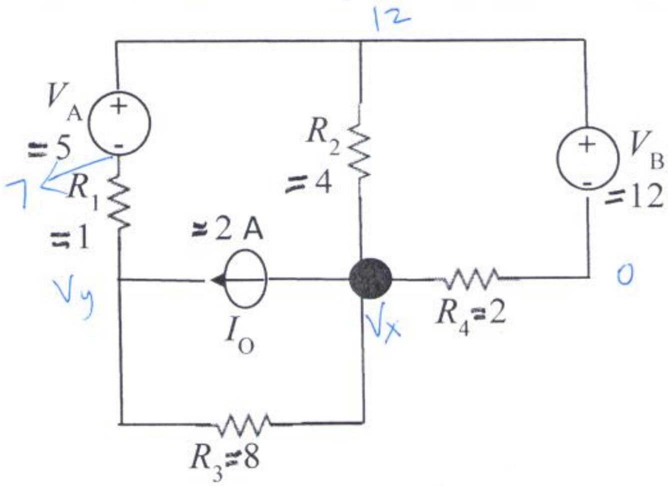
1. **Circuit Analysis (14 points)** Find the current in each element. Show each step in your solution to receive full credit. Two copies of the same schematic are shown in case you need more than one.



$$\begin{cases} \dfrac{7-V_y}{1} + 2 + \dfrac{V_x - V_y}{8} = 0. \\ \dfrac{12-V_x}{4} - \dfrac{V_x}{2} + \dfrac{V_y - V_x}{8} = 2 \end{cases}$$

① : $56 - 8V_y + 16 + V_x - V_y = 0$

$V_x - 9V_y = -72$

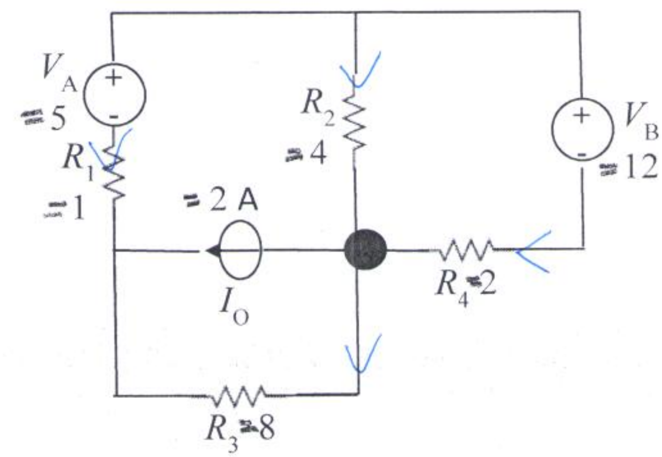
② : $24 - 2V_x - 4V_x + V_y - V_x = 16$

$-7V_x + V_y = -8$

$-62V_x = -144$

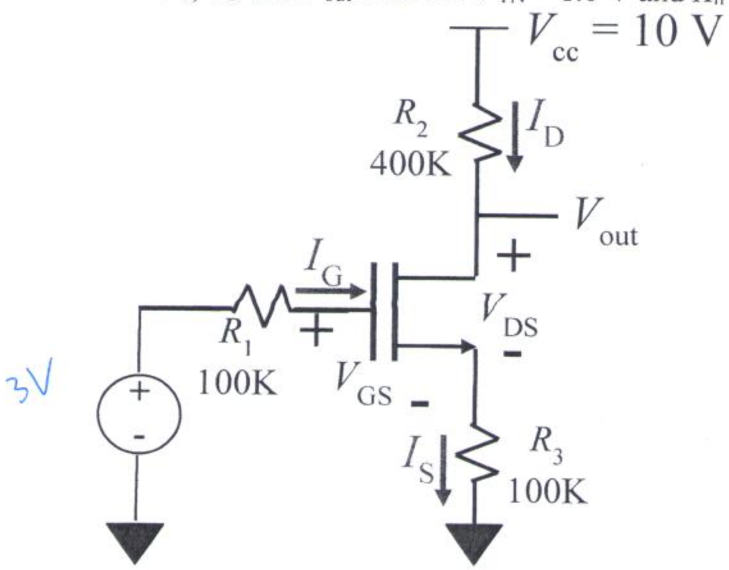
$V_x = \frac{72}{31} = 2.32$

$V_y = 8.26$



Element	Current (A)
I_{R1}	$7 - V_y = -1.26$ A
I_{R2}	$\frac{12-V_x}{4} = 2.42$ A
I_{R3}	$\frac{V_x - V_y}{8} = -0.74$ A
I_{R4}	$\frac{-V_x}{2} = -1.16$ A
I_{VA}	$= I_{R_1}$
I_{VB}	$= I_{R4}$
I_{I0}	2A

2. MOSFET DC Bias (18 points). For the circuit below find the DC values of I_G, I_D, V_G, V_S, V_D and V_{out}. Assume $V_{TN} = 1.0$ V and $K_n = 40\ \mu A/V^2$.



Assume in saturation:

$$I_D = I_S = \frac{k_n}{2}(V_{GS} - V_{th})^2$$

$$= 20 \cdot 10^{-6}(3 - V_S - 1)^2$$

$$I_S \cdot 100k = V_S.$$

$$V_S \cdot \cancel{10^5} = \cancel{20 \cdot 10^{-6}} 2(2 - V_S)^2$$

$$V_S = 2(4 - \cancel{4}V_S + V_S^2)$$

$$2V_S^2 - 9V_S + 8 = 0$$

$$V_S = \frac{9 \pm \sqrt{17}}{4} = \begin{cases} 1.22\text{ V} \\ 3.28\text{ V} \quad \times \end{cases}$$

$V_S = 1.22$V $\quad I_S = 12.2\ uA.$

$V_{GS} = 1.78$V

$V_D = 10V - 400k \cdot 12.2uA$

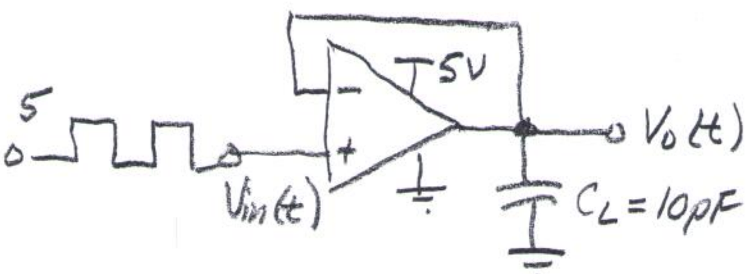
$= 5.12V \quad V_{DS} = 3.9V$

$> V_{GS} - V_{th}.$

$\therefore$ in saturation

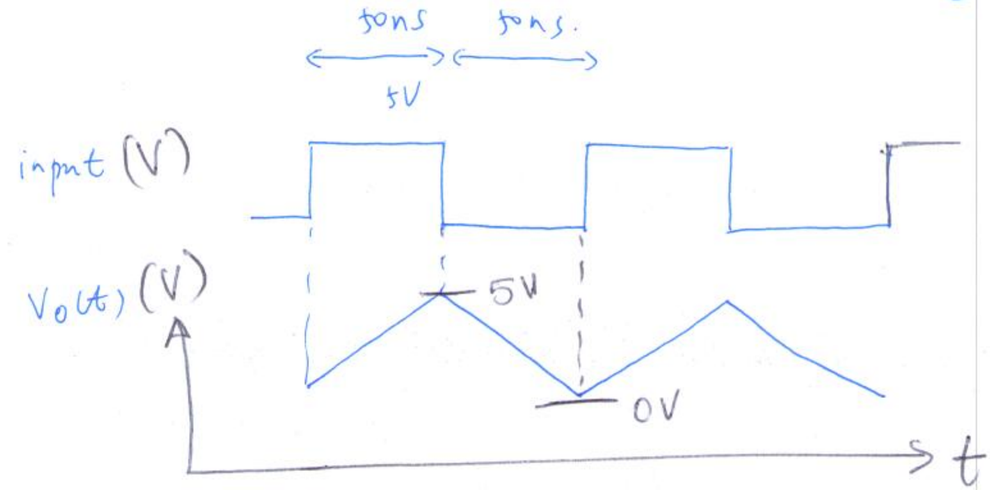
Parameter	Value (w / units)
I_G	0
I_D	12.2 uA
V_G	3V
V_S	1.22V
V_D	5.12V
V_{out}	5.12V

3. Time-Domain Analysis [15 points]. An opamp with rail-to-rail output voltage swing is connected to a 5 V supply. The opamp can supply or sink a maximum of 1 mA of current at its output terminal. It is driving a 10 pF load capacitor as shown. The input is driven with an ideal square wave, $V_{in}(t)$, that goes from 0 to 5V at a frequency of 10MHz. Plot the output voltage waveform, $V_o(t)$.



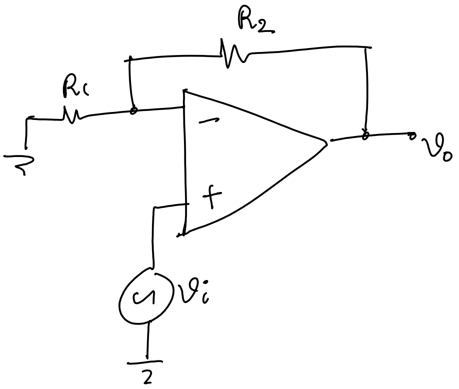
$$V_o(t) = \frac{I \cdot \Delta t}{C_L} = \frac{1m}{10p}\Delta t = 10^8 \Delta t.$$

$$\frac{5V}{10^8} = 50ns.$$



Frequency response

$$A(s) = \frac{A_0}{(1+s/\omega_{p_1})(1+s/\omega_{p_2})}$$



$$(v_+ - v_-)A(s) = v_o$$

$$\frac{v_- - v_o}{R_2} + \frac{v_-}{R_1} = 0$$

$$v_- = \frac{R_1}{R_1+R_2} v_o \qquad v_- = \beta v_o$$

$$\beta = \frac{R_1}{R_1+R_2}$$

$$\Rightarrow \quad (v_i - \beta v_o)A(s) = v_o$$

$$\frac{v_o}{v_i} = \frac{A(s)}{1+\beta A(s)}$$

$$\text{loop gain} = A\beta = \frac{A_0\beta}{(1+s/\omega_{p_1})(1+s/\omega_{p_2})}$$

Since DC gain $= 10^5$ $\quad \omega_{p_1} = 200\pi$ rad/sec $\quad \omega_{p_2} = 4\times10^7\pi$ rad/sec

$$\omega_{p_1}\cdot A_{DC} = 2\times10^7\pi \text{ rad/sec} < \omega_{p_2}.$$

$\omega_u \rightarrow$ At the unity gain frequency $\approx (1+A\beta)\omega_{p_1} \gg \omega_{p_1}$ the phase introduced by first pole $(\omega_{p_1}) \approx 90°$

$$-\tan^{-1}\left(\frac{\omega_u}{\omega_{p_1}}\right) - \tan^{-1}\left(\frac{\omega_u}{\omega_{p_2}}\right) = -100°$$

$$-90 - \tan^{-1}\left(\frac{\omega_u}{\omega_{p_2}}\right) = -100°$$

$$\boxed{\omega_u = \omega_{p_2}\tan 10°}$$

The magnitude of loop gain = 1 @ ω_u

$$\left| \frac{A_o \beta}{\left(1 + \frac{j\omega_u}{\omega_{p1}}\right)\left(1 + \frac{j\omega_u}{\omega_{p2}}\right)} \right| = 1$$

ω_u

$$\frac{10^5 (\beta)}{\frac{\omega_{p2} \tan 10^\circ}{\omega_{p1}} \left| \left(1 + j \tan 10^\circ\right) \right|} = 1$$

$$\beta = \frac{\omega_{p2} \tan 10^\circ}{10^5 \omega_{p1}} \left| 1 + j \tan 10^\circ \right|$$

$$\frac{R_1}{R_1 + R_2} = \beta$$

$$\beta = \frac{\cancel{4\pi} \times \cancel{10^7}}{\cancel{2\pi} \times 100 \times \cancel{10^5}} \cdot \tan 10^\circ \left| 1 + j \tan 10^\circ \right|$$

$$\frac{R_2}{R_1 + R_1} = 1 - \beta$$

$$\beta = 2 \tan 10^\circ \left| 1 + j \tan 10^\circ \right|$$

$$\frac{R_2}{R_1} = \frac{(1-\beta)}{\beta}$$

$$\beta = 0.36$$

$$\boxed{R_2 = \frac{(1-\beta) R_1}{\beta}}$$

$R_1 = 1 K\Omega$

$R_2 =$ 1.79 KOhm

(b). $V_o(s) = V_i(s) \cdot H(s)$ $\qquad V_i(s) = 1/s$

$$H(s) = \frac{A(s)}{1+\beta A(s)} = \frac{\dfrac{A_o}{(1+s/\omega_{p_1})(1+s/\omega_{p_2})}}{1+\dfrac{\beta A_o}{(1+s/\omega_{p_1})(1+s/\omega_{p_2})}}$$

$$= \frac{A_o}{(A_o\beta) + \left(1+\frac{s}{\omega_{p_1}}\right)(1+s/\omega_{p_2})}$$

Multiply by $\omega_{p_1}\omega_{p_2}$

$$= \frac{A_o\,\omega_{p_1}\omega_{p_2}}{(A_o\beta)\,\omega_{p_1}\omega_{p_2} + (s+\omega_{p_1})(s+\omega_{p_2})}$$

$\omega_{p_1} = 2\pi \times 10^2$ rad/s

$\omega_{p_2} = 4\pi \times 10^7$ rad/s

$$= \frac{A_o\,\omega_{p_1}\omega_{p_2}}{s^2 + s(\omega_{p_1}+\omega_{p_2}) + \omega_{p_1}\omega_{p_2}(1+A_o\beta)}$$

$A_o = 10^5$

$\beta = \frac{R_1}{R_1+R_2}$

$\beta = \frac{1}{5}$

$$H(s) = \frac{A_o\,\omega_{p_1}\,\omega_{p_2}}{(s+a)(s+b)}$$

$a = 1.416 \times 10^7$ rad/sec

$b = 11.1 \times 10^7$ rad/sec

$$V_o(s) = \frac{A_o\,\omega_{p_1}\omega_{p_2}}{s(s+a)(s+b)} = \frac{K_1}{s} + \frac{K_2}{s+a} + \frac{K_3}{s+b}$$

$$A_o\omega_{p_1}\omega_{p_2} = K_1(s+a)(s+b) + K_2 s(s+b) + K_3 s(s+a)$$

Set $s=0$ $\qquad K_1 = \dfrac{A_o\omega_{p_1}\omega_{p_2}}{ab}$ $\qquad s=-a$: $K_2 = -\dfrac{A_o\omega_{p_1}\omega_{p_2}}{a(b-a)}$ $\qquad s=-b$: $K_3 = -\dfrac{A_o\omega_{p_1}\omega_{p_2}}{b(a-b)}$

Taking inverse laplace transform

$$V_o(t) = k_1 u(t) + k_2 e^{-at} u(t) + k_3 e^{-bt} u(t)$$

$$V_o(t) = A_0 \omega_{p1} \omega_{p2} \left(\frac{1}{ab} - \frac{e^{-at}}{a(b-a)} - \frac{e^{-bt}}{b(a-b)} \right) u(t)$$

$A_0 = 10^5$ $\omega_{p1} = 200\pi$ rad/sec

$a = 1.416 \times 10^7$ rad/Sec $\omega_{p2} = 4\pi \times 10^7$ rad/sec

$b = 11.15 \times 10^7$ rad/sec

$$V_o(t) = 5 - 5.728\, e^{-at} + 0.727\, e^{-bt}$$

See that gain is 5 if $t \to \infty$