Physics 7B, Speliotopoulos
Final Exam, Fall 2014
Berkeley, CA

Rules: *This final exam is closed book and closed notes. In particular, calculators are not allowed during this exam. Cell phones must be turned off during the exam, and placed in your backpacks, or bags.* ***They cannot be on your person.***

Please make sure that you do the following during the midterm:

- ***Show all your work in your blue book***
- *Write your name, discussion number, ID number on all documents you hand in.*
- *Make sure that the grader knows what s/he should grade by circling your final answer.*
- *Cross out any parts of your solutions that you do not want the grader to grade.*

Each problem is worth 20 points. We will give partial credit on this final*, so if you are not altogether sure how to do a problem, or if you do not have time to complete a problem, be sure to write down as much information as you can on the problem. This includes any or all of the following: Drawing a clear diagram of the problem, telling us how you would do the problem if you had the time, telling us why you believe (in terms of physics) the answer you got to a problem is incorrect, and telling us how you would mathematically solve an equation or set of equations once the physics is given and the equations have been derived. Don't get too bogged down in the mathematics; we are looking to see how much physics you know, not how well you can solve math problems.*

If at any point in the exam you have any questions, just raise your hand, and we will see if we are able to answer them.

Copy and fill in the following information on the front of your bluebook:

Name: ______________________________ *Disc Sec Number:* _______________________

Signature: ___________________________ *Disc Sec GSI:* __________________________

Student ID Number: ____________________

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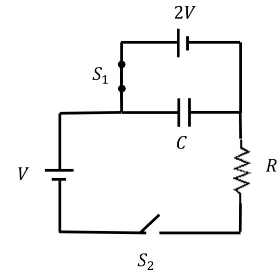
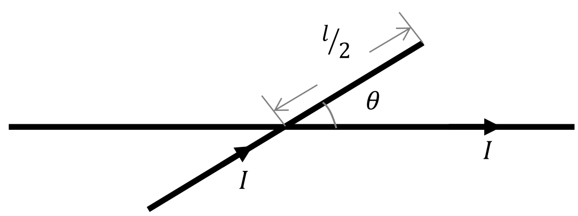
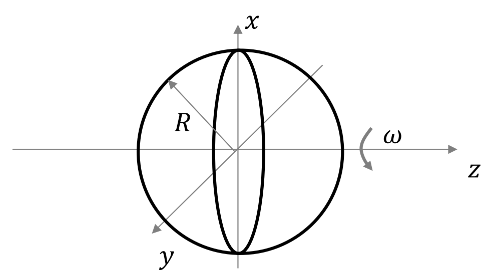
1. The RC circuit on the right has a capacitor, C, a resistor, R, voltage supplies $2V$ and V, along with a switch S_1 that is closed, and a switch S_2 that is open. At $t = 0$ the switch S_1 is opened and the switch S_2 closed.
   a. What is the charge, $q(0)$, on the capacitor at $t = 0$? Express it in terms of C, and V.
   b. What is the charge, $q(t)$, on the capacitor for $t > 0$? Express it in terms of R, C, and V.



2. The figure below shows an infinitely long wire with a current, I, flowing through it. Across this wire lies a second wire with length, l, and current I flowing through it as well; the infinitely long wire crosses the finite wire at its midpoint. What is the torque on the second wire? Express it in terms of I, μ_0, and θ. Assume that the wires are insulated, and have negligible thickness.



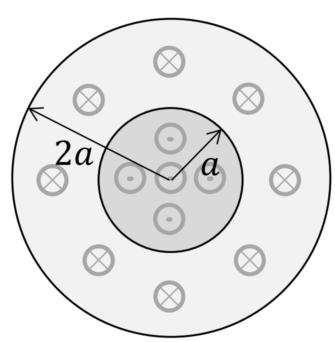
3. The figure to the right shows a ***hollow*** sphere with radius, R, total charge, Q. It is spinning along the z-axis with angular velocity, ω. What is the magnetic field at the center of the sphere (magnitude and direction)? Express it in terms of Q, R, ω, and μ_0.



4. The figure on the right shows two concentric circles, one with radius, a, and the second with radius, $2a$. Within the inner circle there is a magnetic field, $B_I(t)$, while between the inner and outer circles there is a magnetic field, $B_{II}(t)$. Both fields oscillate with time as

$$B_I(t) = B_0 \cos(\omega t), \qquad B_{II}(t) = -\frac{B_0}{3}\cos(\omega t),$$

and they oscillate into and out of the paper. (At the time, t, shown in the figure, $B_I(t)$ points out of the paper and $B_{II}(t)$ points into the paper.) What is the electric field in the following regions: *i*) $r \le a$, *ii*) $a < r \le 2a$, and *iii*) $2a < r$? Express it in terms of any or all of the following variables: t, ω, B_0, r, and a.



5. Figure A to the right shows an inductor made from two sheets of current each with width, w, and length, l, and they are separated by a distance, d. The left sheet has a current per unit length, j, flowing out of the page, while the right has the same, j, flowing into the page. The thickness of the sheets is negligible, and $d \ll l$ and $d \ll w$ so that the sheets can be treated as infinite.
   a. What is the self inductance, L, of the inductor? Express it in terms of d, w, l, and μ_0.
   b. In Fig B, two sheets of metal with negligible thickness are placed in the inductor. If j changes with time, an emf is measured between the two metal sheets. What is the mutual inductance, M, of the system? Express it in terms of x, w, l, and μ_0.

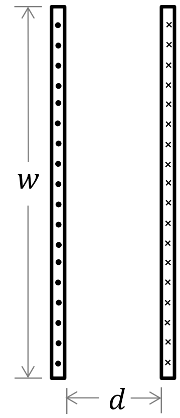


Figure A

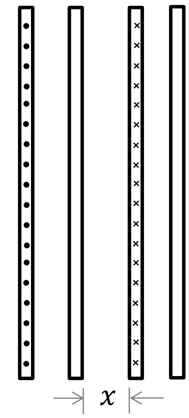
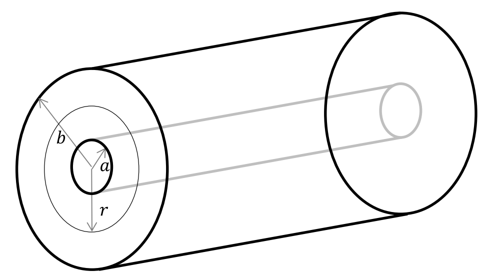


Figure B

6. The elastance, Y, of a capacitor with capacitance, C, is $Y = \frac{1}{C}$. It is often easier to use when two capacitors, $C_1 = \frac{1}{Y_1}$ and $C_2 = \frac{1}{Y_2}$, are in series since the equivalent elastance of the two capacitors is simply $Y = Y_1 + Y_2$.

   The figure to the right shows a ***cylindrical*** capacitor with inner radius, a, and outer radius, b. It is filled with a dielectric with a dielectric constant that varies radially as
$$K(r) = K_0 \left(\frac{a}{r}\right)^2.$$
   What is the capacitance per unit length, C/l, of the capacitor? (I would recommend you find the equivalent elastance first.)



7. A material contains, N, atoms at a temperature, T. The valence electron for the atom can be in one of two energy levels, one with energy $E_1 = E_0$, and the other with energy, $E_2 = E_0 + \Lambda$. Because there are only two possible energies an atom can have, the average of any function, $G(E)$, of the energy, E, of the electron is calculated using a simple sum instead of an integral,
$$\langle G(E) \rangle = A \sum_{n=1}^{2} G(E_n)\, e^{-\frac{E_n}{k_B T}},$$
   where A is a normalization constant and k_B is the Boltzmann constant.
   a. Using the fact that $\langle 1 \rangle = 1$, what is A? Express it in terms of E_0, Λ, k_B, and T.
   b. What is the average energy, $\langle E \rangle$, of a valence electron? Express it in terms of E_0, Λ, k_B, and T.
   c. What is $\langle E \rangle$ when $T \longrightarrow \infty$ and when $T \longrightarrow 0$?

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\Delta l = \alpha l_0 \Delta T$$

$$\Delta V = \beta V_0 \Delta T$$

$$PV = NkT = nRT$$

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

$$f_{Maxwell}(v) = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}$$

$$E = \frac{D}{2}nRT$$

$$Q = mc\Delta T = nC\Delta T$$

$$Q = mL \text{ (For a phase transition)}$$

$$dE = -PdV + dQ$$

$$W = \int PdV$$

$$C_P - C_V = R = N_A k$$

$$PV^\gamma = \text{const. (For an adiabatic process)}$$

$$\gamma = \frac{C_P}{C_V} = \frac{d+2}{d}$$

$$C_V = \frac{d}{2}R$$

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

$$e = \frac{W}{Q_h}$$

$$e_{ideal} = 1 - \frac{T_L}{T_H}$$

$$dQ = TdS$$

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$\vec{E} = \int \frac{dQ}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r} \text{ (For infinite wire)}$$

$$\rho = \frac{dQ}{dV}$$

$$\sigma = \frac{dQ}{dA}$$

$$\lambda = \frac{dQ}{dl}$$

$$\vec{p} = Q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\Delta U = Q\Delta V$$

$$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$V = \int \frac{dQ}{4\pi\epsilon_0 r}$$

$$\vec{E} = -\vec{\nabla} V$$

$$Q = CV$$

$$C_{eq} = C_1 + C_2 \text{ (In parallel)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \text{ (In series)}$$

$$C = \frac{\epsilon A}{d} \text{ (parallel plate)}$$

$$C = \frac{2\pi\epsilon l}{\ln(r_a/r_b)} \text{ (cylindrical)}$$

$$C = 4\pi\epsilon\frac{r_a r_b}{r_a - r_b} \text{ (spherical)}$$

$$\epsilon = \kappa\epsilon_0$$

$$U = \frac{Q^2}{2C}$$

$$U = \int \frac{\epsilon_0}{2}|\vec{E}|^2 dV$$

$$I = \frac{dQ}{dt}$$

$$\Delta V = IR$$

$$R = \rho\frac{l}{A}$$

$$\rho(T) = \rho(T_0)(1 + \alpha(T - T_0))$$

$$P = IV$$

$$I = \int \vec{j} \cdot d\vec{A}$$

$$\vec{j} = nQ\vec{v_d} = \frac{\vec{E}}{\rho}$$

$$R_{eq} = R_1 + R_2 \text{ (In series)}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \text{ (In parallel)}$$

$$\sum_{\text{junc.}} I = 0 \text{ (junction rule)}$$

$$\sum_{\text{loop}} V = 0 \text{ (loop rule)}$$

$$d\vec{F}_m = Id\vec{l} \times \vec{B}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{\mu} = NI\vec{A}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu} \cdot \vec{B}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{Id\vec{l} \times \hat{r}}{r^2}$$

$$|\vec{B}| = \frac{\mu_0 I}{2\pi r} \text{ (For infinite wire)}$$

$$|\vec{B}| = \frac{\mu_0 I}{2} \frac{R^2}{(z^2 + R^2)^{3/2}}$$

(Current loop of radius R)

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$M = N_1 \frac{\Phi_1}{I_2} = N_2 \frac{\Phi_2}{I_1}$$

$$L = N\frac{\Phi_B}{I}$$

$$U = \frac{1}{2}LI^2$$

$$U = \int \frac{1}{2\mu_0} |\vec{B}|^2 dV$$

$$\overline{g(v)} = \int_0^\infty g(v) \frac{f(v)}{N} dv$$

($f(v)$ a speed distribution)

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial f}{\partial z}\hat{z}$$

$$d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + dz\hat{z}$$

(Cylindrical Coordinates)

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin(\theta)d\phi\hat{\phi}$$

(Spherical Coordinates)

$$y(t) = \frac{B}{A}(1 - e^{-At}) + y(0)e^{-At}$$

$$\text{solves } \frac{dy}{dt} = -Ay + B$$

$$y(t) = y_{max}\cos(\sqrt{A}t + \delta)$$

$$\text{solves } \frac{d^2y}{dt^2} = -Ay$$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n! 2^{2n+1}} \sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int_0^\pi \sin^3(x) dx = \frac{4}{3}$$

$$\int (1 + x^2)^{-1/2} dx = \ln(x + \sqrt{1 + x^2})$$

$$\int (1 + x^2)^{-1} dx = \arctan(x)$$

$$\int (1 + x^2)^{-3/2} dx = \frac{x}{\sqrt{1 + x^2}}$$

$$\int \frac{x}{1 + x^2} dx = \frac{1}{2}\ln(1 + x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\int \sin(x) dx = -\cos(x)$$

$$\int \cos(x) dx = \sin(x)$$

$$\int \frac{dx}{x} = \ln(x)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1 + x)^\alpha \approx 1 + \alpha x + \frac{(\alpha - 1)\alpha}{2}x^2$$

$$\ln(1 + x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$