Physics 7C Section 2
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Choose two out of the three problems in section A and two out of the three problem in section B. The test duration is 180 minutes.

A 1. A particle travelling faster than light in a medium of refractive index n emits coherent radiation, known as Cherenkov radiation. i) Using the Huygens principle to locate the coherent wavefront, show that the radiation is emitted at an angle θ_c, w.r.t. the particle velocity vector, such that $\cos\theta_c = \frac{1}{\beta n}$. ii) Determine the minimum particle speed $\beta = v/c$ which produces Cherenkov radiation, as function of the refractive index n. iii) Determine the minimum energy at which a proton (M_p = 938 MeV) emits Cherenkov radiation in gaseous C_5F_{12} (n = 1.00044). (Hint: consider pulses emitted by the particle at times t_0, t_1, ..., get the distances travelled by the radiation, emitted at an angle θ_c, compare them to the distance travelled by the particle and use this to define the condition for the creation of a coherent wavefront.)

A 2. An experiment is aiming at the production of a particle Z' with mass = 500 GeV/c^2 using electron and positrons collisions through the reaction $e^-e^+ \to Z' \to e^+e^-$. Find the positron energy necessary for producing the Z' particle i) if the positrons collide on electrons at rest in the lab frame, ii) if the positrons collide head-on with electrons of the same energy. iii) In both cases, find the minimum particle energies necessary for the outgoing electron and positron momenta to exceed 10 GeV/c. iv) Determine the radius of curvature of the 10 GeV/c e^- and e^+ trajectories, if a B = 1 T solenoidal magnetic field is applied.

A 3. The energy spectrum of cosmic rays has an upper bound, due to their scattering against the Cosmic Microwave Backgrond photons, known as Greisen-Zatsepin-Kuzmin (GZK) cut-off. Use a simple model, where the scattering is due to head-on collision and the cutoff is defined by the reaction $p\gamma \to p\pi$, (M_p = 938 MeV/c^2, M_π = 139 MeV/c^2, $E_\gamma = kT = 3 \times 10^{-10}$ MeV) becoming kinematically allowed, to estimate the value of the GZK cut-off energy.

B 1. Consider an electron, e^-, in a 1D, infinite potential well of length L. Due to the uncertainty principle its velocity v is non-zero, if L is finite. Find the value of L for which the electron would become relativistic, taking this to correspond to the β value at which its relativistic mass exceeds the rest mass, M_e = 0.511 MeV/c^2, by more than 5 %.

B 2. The fine splitting of the Hydrogen atomic energy levels is caused by the interaction of the spin of the electron with its orbital motion. i) Give the possible values of the spin magnetic moment of the electron μ_z in units of the Bohr magneton, μ_B = 5.79 $\times 10^{-5}$ eV T^{-1} and the expression of the total potential energy of the electron, ii) determine the strength of the magnetic field B, caused by the apparent motion of the proton around the electron, knowing that the splitting of the $2P$ Hydrogen energy level is ΔE = 4.5 $\times 10^{-5}$ eV. iii) determine whether the J = 3/2 or the J = 1/2 state has the larger energy and justify why.

B 3. An electron, e^-, rotating around an anti-electron, or positron e^+, forms a bound system, called Positronium. i) Write the potential $U(r)$ felt by the electron and its radial Schrodinger equation, ii) write the positronium 1S wave function and iii) find the ℓ=0 energy levels, taking the electron to be non-relativistic. (Hints: since the mass of the "nucleus" here is equal to that of the electron M_e = 0.511 MeV/c^2, its kinetic energy must be taken into account. In the case of hydrogen $C_{100} = \frac{1}{\sqrt{\pi}} a_0^{-3/2}$, where $a_0 = \frac{\hbar}{m_e k e^2}$=0.053 nm is the Bohr radius. How is this modified in positronium ?)