1)

For this problem, the capacitors act like wires at $t = 0$ (as they are uncharged), while at $t = \infty$ they behave like open circuits (as they are fully charged).

a) This is the scenario for two resistors in parallel.

$I_1 = \frac{V}{R_1}$ $I_2 = \frac{V}{R_2}$

b) Across C_1 and C_2, there is no voltage drop.

$\Rightarrow V_b - V_a = V$

c) There is no current flowing through the circuit, and hence there is no voltage drop across R_1 and R_2. $V_b - V_a = -V$

d) When switch 2 is closed, at equilibrium current flows only through the resistors. Since R_1 and R_2 are now in series,

$V_b - V_{ground} = V\frac{R_1}{R_1+R_2}$

e) Before switch 2 was closed, the charges on the two capacitors are given by:

$Q_1 = C_1V$ $Q_2 = C_2V$

After switch 2 was closed, the capacitors C_1, C_2 are in parallel with the resistors R_2, R_1 respectively. The charges are given by:

$Q_1\prime = C_1V\frac{R_2}{R_1+R_2}$ $Q_2\prime = C_2V\frac{R_1}{R_1+R_2}$

The changes in charges are given by:

$\Delta Q_1 = Q_1\prime - Q_1 = -C_1V\frac{R_1}{R_1+R_2}$

$\Delta Q_2 = Q_2\prime - Q_2 = -C_2V\frac{R_2}{R_1+R_2}$

2)

Let $\mathbf{r}$ denote the radial vector pointing outwards from the center of the spherical shells.

a) First, slice the region into concentric spherical shells of thickness dr. Note that these shells together constitute resistors in series.

$R = \int dR = \int \rho \frac{dl}{A} = \int_a^b \rho \frac{dr}{4\pi r^2} = \frac{\rho}{4\pi}(\frac{1}{a} - \frac{1}{b})$

b) Take $A = 4\pi b^2$. (One can also use $A = 4\pi a^2$)

Taylor expanding the expression from a),

$R = \frac{\rho}{4\pi}\frac{b-a}{ab} = \frac{\rho}{4\pi}\frac{L}{b(b-L)} = \frac{\rho L}{4\pi b^2}(1 - \frac{L}{b})^{-1} \cong \frac{\rho L}{4\pi b^2} = \frac{\rho L}{A}$

c) $\mathbf{j}(r) = \frac{\mathbf{I}}{4\pi r^2} = -\frac{V_{ab}}{\rho}\frac{ab}{b-a}\frac{1}{r^2}\hat{r}$

Solution 3:

a. Since both portions of the capacitor (one without dielectric and the other with dielectric) would experience the same potential difference when connected in a circuit, the given capacitor can be considered as a combination of 2 capacitors in parallel.

One of these capacitors has no dielectric and has height (L-h).
$C_1 = A\varepsilon/d = L*(L-h)/d$

The other capacitor has fuel as dielectric and height h.
$C_2 = K*L*h/d$

$C_{eq} = C1+C2$

$C_{eq} = L/d\ [L-h+Kh]$ ……………………………………………(1)

b. Now if fuel of dielectric constant ($\boldsymbol{K} + \boldsymbol{\xi}$) is used, the C_{eq} would change to:

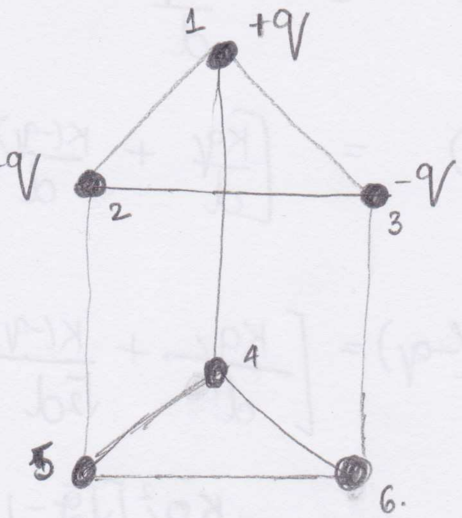
$C_{eq}' = = L/d\ [L-h+(\boldsymbol{K} + \boldsymbol{\xi})h]$………………………………..(2)

Since fuel gauge uses the value of capacitance to measure the fuel level and $C_{eq}' > C_{eq}$, the gauge will show more fuel level than actually present.

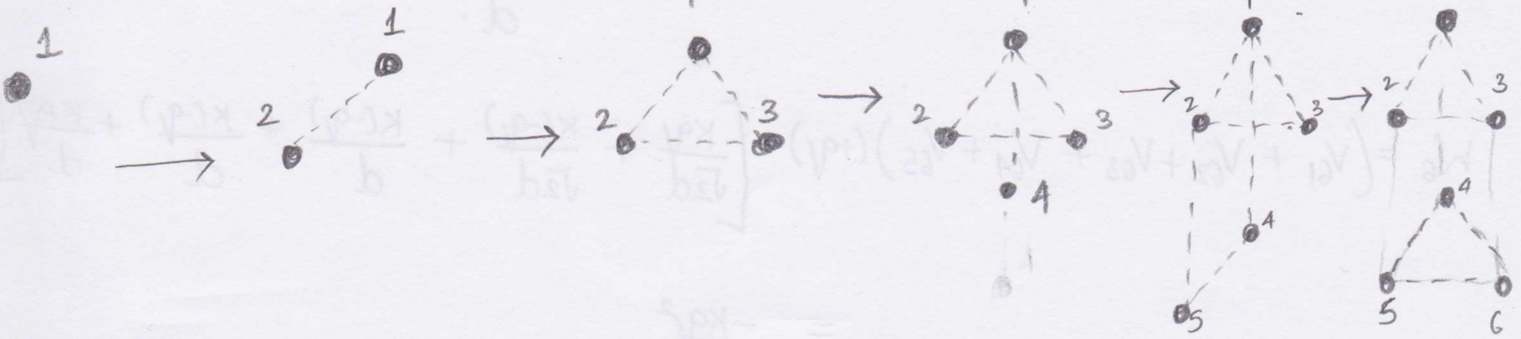
% error can be expressed as $(C_{eq}' - C_{eq})*100/ C_{eq}$

% error= $L\ \xi\ h/K$

Solution 4.



Assuming, initially all the charges are at infinite separation $(U_i = 0)$ and then we add one charge at a time to form this configuration in the sequence labelled in the figure.



As we keep on adding charges, we are doing work against the potential of existing charges and total work done = final P.E.

$W_n \rightarrow$ Work done in bringing n^{th} charge to configuration (in existing potential of $(n-1)$ charges.

$V_{ij} \rightarrow$ Potential at position of charge 'i' due to charge 'j']

Thus, $U_f = W_1 + W_2 + W_3 + W_4 + W_5 + W_6$

$W_1 = 0$ (No potential change as charge 1 is moved from infinity to this point)

$W_2 = V_{21}\, q_2$ $[V_{21} \rightarrow$ Potential at position of 2 because of 1$]$

$$W_2 = \frac{Kq}{d}(-q) = -\frac{Kq^2}{d}$$

$$W_3 = (V_{31} + V_{32})(-q) = \left[\frac{Kq}{d} + \frac{K(-q)}{d}\right](-q) = 0$$

$$W_4 = (V_{41} + V_{42} + V_{43})(-q) = \left[\frac{Kq}{d} + \frac{K(-q)}{\sqrt{2}d} + \frac{K(-q)}{\sqrt{2}d}\right](-q)$$

$$= -\frac{Kq^2}{d}[\sqrt{2}-1]$$

$$W_5 = (V_{51} + V_{52} + V_{53} + V_{54})(+q) = \left[\frac{Kq}{\sqrt{2}d} + \frac{K(-q)}{d} + \frac{K(-q)}{\sqrt{2}d} + \frac{K(-q)}{d}\right](+q)$$

$$= \frac{-2Kq^2}{d}$$

$$W_6 = (V_{61} + V_{62} + V_{63} + V_{64} + V_{65})(+q) = \left[\frac{Kq}{\sqrt{2}d} + \frac{K(-q)}{\sqrt{2}d} + \frac{K(-q)}{d} + \frac{K(-q)}{d} + \frac{Kq}{d}\right](+q)$$

$$= \frac{-Kq^2}{d}$$

$$U_f = -\frac{Kq^2}{d}\left[5-\sqrt{2}\right] \Rightarrow U_f < 0$$

Since PE of the system is negative, this means that the system (ionic crystal) is stable.

Problem 5

Due to the cylindrical geometry, we use Gauss's law to solve this problem. The problem is equivalent to a cylinder with charge density $+\rho$ with radius R centered at O superimposed with a cylinder with charge density $-\rho$ with radius r centered at d. Then, in the inner cylinder the net charge density is 0, giving us the scenario described in the problem.
First consider the cylinder with charge density $+\rho$ with radius R centered at O. Take the coordinates of the cylindrical coordinate system to be (ξ, θ, z). As the system is rotationally symmetric, if we are at a point ξ, it should not depend what θ we are at, since no matter what θ is, the system will look the same. For the same reason, the field cannot depend on the z coordinate. Thus, $|\vec{E}_+|$ is only a function of ξ.
Since the cylinder is infinite, the charge distribution looks the same in both directions in z. Thus, there can be no force acting in the z direction. Similarly, if we draw a line connecting the location of a charge and the center, there will be as much charge to the left as to the right of that line, so there cannot be a force in the θ direction. Thus, $\vec{E}_+$ only point in the ξ direction.
Choose a Gaussian surface that is a cylinder with center O, has radius ξ, and length l. As argued before, the electric field points radially, so the endcaps of the cylinder do not contribute to the Gauss's law integral, as the area normal of the endcaps point in the $\hat{z}$ direction, and $\hat{z} \cdot \hat{\xi} = 0$. On the rest of the cylinder, the area normal vector points radially outward. Thus for points $\xi < R$,

$$\oint \vec{E}_+(\xi < R) \cdot d\vec{A} = |\vec{E}_+(\xi < R)| 2\pi \xi l = \frac{q_{encl}}{\epsilon_0} = \frac{\rho \pi \xi^2 l}{\epsilon_0}$$

Which gives

$$\vec{E}_+(\xi < R) = \frac{\rho}{2\epsilon_0} \vec{\xi}$$

By a similar argument, we get that the field due to the negative charge density (inside the negative charge density region) is (accounting for the proper shift in coordinates)

$$\vec{E}_-(|\vec{\xi} - \vec{d}| < r) = -\frac{\rho}{2\epsilon_0}(\vec{\xi} - \vec{d})$$

So the total field is

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{\rho}{2\epsilon_0}(\vec{\xi} - (\vec{\xi} - \vec{d})) = \frac{\rho}{2\epsilon_0}\vec{d} = \frac{\rho d}{2\epsilon_0}\hat{x} = \frac{\rho d}{2\epsilon_0}(\cos(\theta)\hat{r} - \sin(\theta)\hat{\theta})$$