PHYSICS 7B, Lectures 1 & 3 – Spring 2015
Midterm 2, C. Bordel
Monday, April 6, 2015
7pm-9pm

Make sure you show all your work and justify your answers in order to get full credit.

<u>Problem 1</u> – Resistance & current (10 pts)

Two cylindrical wires of respective resistivities ρ_1 and ρ_2, respective lengths ℓ_1 and ℓ_2, and same diameter d are connected to each other, as shown in Figure 1a.

a) Calculate the resistance R of the 2 connected wires made of different materials.

This can be used to monitor the level of liquid helium in a storage tank. A niobium-titanium (Nb-Ti) wire of length ℓ spans the entire height of the tank, and an electronic circuit maintains a constant electrical current at all times in the wire. A voltmeter monitors the voltage V across the wire, as shown in Figure 1b. Because Nb-Ti is superconducting at low temperatures, the portion of the wire immersed in liquid helium (length x) therefore has zero resistivity. The portion above the liquid always has nonzero resistivity ρ.

b) Calculate the ratio V/V_0, where V_0 is the voltage measured when the tank is empty. Give your answer in terms of the fraction f of the tank which is filled with liquid helium.

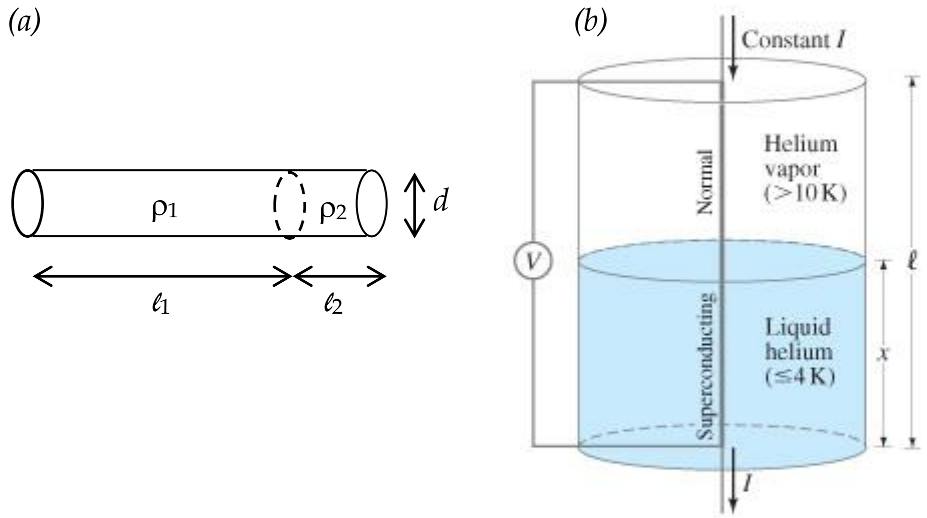

Figure 1

<u>Problem 2</u> – DC circuit (15 pts)

Two resistors of resistance R and two capacitors of capacitance C are combined as shown in Figure 2 to form a circuit, where the battery sources a voltage $\mathcal{E}$.

a) Draw a simplified version of that electrical circuit using only one resistor of equivalent resistance R_{eq} and one capacitor of equivalent capacitance C_{eq}. Express R_{eq} and C_{eq} as a function of R and C.

b) Before the battery is connected to the circuit, the capacitors are uncharged. Establish the differential equation satisfied by the charge Q accumulating on the equivalent capacitor's plates, using R_{eq} and C_{eq}. *You don't need to solve the equation!*

c) Determine, without any calculation, the current I going through the equivalent circuit immediately after the battery is connected to the circuit, and then after an infinite amount of time.

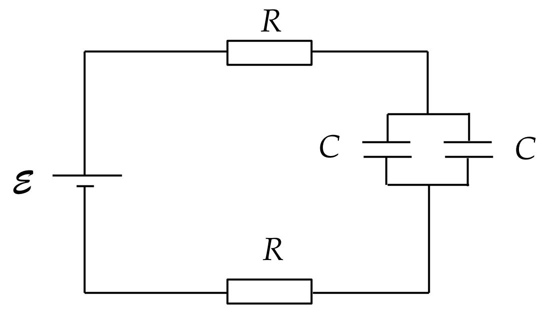

Figure 2

Problem 3– Capacitor & dielectric (20 pts)

Two coaxial cylindrical conducting shells of identical length L and respective radii R_1 and R_2 ($R_2>R_1$) carry uniformly distributed electric charge $+Q$ and $-Q$ respectively ($Q>0$). They are separated by a vacuum of permittivity ε_0.
You may assume that $L>> R_2$. A cross-section view is presented in Figure 3a.

a) Calculate the electric field created at any point M located at a radial distance r from the symmetry axis. *Show your work!*
b) Calculate the absolute value of the voltage between the 2 conducting shells.
c) Determine the capacitance of this cylindrical capacitor.
d) Calculate the capacitance if the gap between the 2 shells is filled by 2 successive dielectric materials of dielectric constant K_1 and thickness d for the inner one, and dielectric constant K_2 for the outer one, as sketched in Figure 3b.

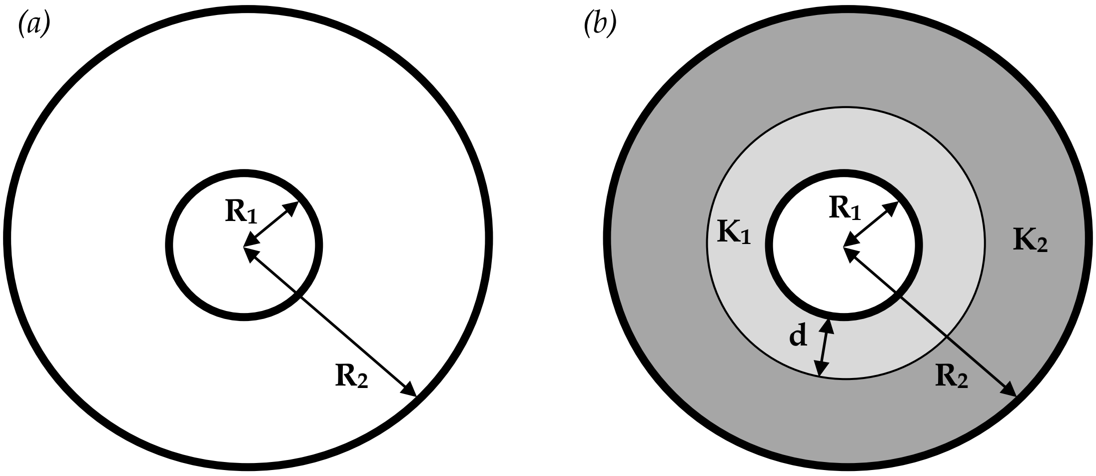

Figure 3: cross-section views

Problem 4 – Electric potential & potential energy (20 pts)

An electron of mass m and electric charge $-e$, initially at rest, is released from infinity along the symmetry axis of a uniformly charged disk of radius R. The flat disk carries positive surface charge distribution σ. Calculate the kinetic energy of the electron at distance z from the center of the disk (Fig.4). You may assume that the gravitational potential energy is negligible.

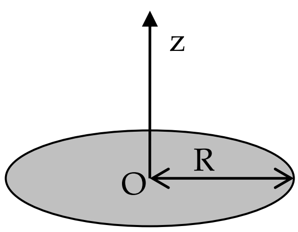

Figure 4

<u>Problem 5</u> – Electric field & potential (25 pts)

We assume that atoms can be modeled by considering a spherical <u>negative</u> charge distribution $\rho(r)$ extending beyond the radial distance a (a >0), around the nucleus of charge q (q>0).
At distance r from the center O of the atom, the electric potential is given by the following expression:

$$V(r) = \frac{q}{4\pi\varepsilon_0 r} e^{-r/a}$$

a) Determine the direction and magnitude of the electric field $\vec{E}(r)$ created by this charge distribution at a distance r from the origin.
b) Calculate the flux $\Phi_E(r)$ of the electric field through a sphere of center O and radius r.
c) Calculate the electric charge Q_{tot} enclosed in the sphere of center O and radius r. What is the limit of Q_{tot} when $r \to \infty$? Interpret your result.
d) Determine the negative charge distribution $\rho(r)$.
Hint: it might be helpful to express the infinitesimal amount of negative charge dQ contained in a spherical shell of thickness dr.

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\hat{r} = \frac{kQ_1Q_2}{r^2}\hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$\vec{E} = \int \frac{dQ}{4\pi\epsilon_0 r^2}\hat{r} = \int \frac{kdQ}{r^2}\hat{r}$$

$$\rho = \frac{dQ}{dV}$$

$$\sigma = \frac{dQ}{dA}$$

$$\lambda = \frac{dQ}{dl}$$

$$\vec{p} = Q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon}$$

$$\Delta U = Q\Delta V$$

$$V = -\int \vec{E} \cdot d\vec{l}$$

$$V = \int \frac{dQ}{4\pi\epsilon_0 r} = \int \frac{kdQ}{r}$$

$$\vec{E} = -\vec{\nabla} V$$

$$Q = CV$$

$$C_{eq} = C_1 + C_2 \text{ (In parallel)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \text{ (In series)}$$

$$\epsilon = \kappa\epsilon_0$$

$$C = \kappa C_0$$

$$U = \frac{Q^2}{2C}$$

$$U = \int \frac{\epsilon}{2}|\vec{E}|^2 dV$$

$$I = \frac{dQ}{dt}$$

$$\Delta V = IR$$

$$R = \rho\frac{l}{A}$$

$$\rho(T) = \rho(T_0)(1 + \alpha(T - T_0))$$

$$P = IV$$

$$I = \int \vec{j} \cdot d\vec{A}$$

$$\vec{j} = nQ\vec{v}_d = \frac{\vec{E}}{\rho}$$

$$R_{eq} = R_1 + R_2 \text{ (In series)}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \text{ (In parallel)}$$

$$\sum_{\text{junction}} I = 0$$

$$\sum_{\text{loop}} V = 0$$

$$y(t) = \frac{B}{A}(1 - e^{-At}) + y(0)e^{-At}$$
$$\text{solves } \frac{dy}{dt} = -Ay + B$$

$$y(t) = y_{max}\cos(\sqrt{A}t + \delta)$$
$$\text{solves } \frac{d^2y}{dt^2} = -Ay$$

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial f}{\partial z}\hat{z} \\ d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + dz\hat{z} \end{aligned}$$
(Cylindrical Coordinates)

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial f}{\partial \phi}\hat{\phi} \\ d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + r\sin(\theta)d\phi\hat{\phi} \end{aligned}$$
(Spherical Coordinates)

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n! 2^{2n+1}}\sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int (1+x^2)^{-1/2} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{x}{\sqrt{1+x^2}} dx = \sqrt{1+x^2}$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha - 1)\alpha}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$